Introduction, Carbon Dioxide (CO_2)

Because of the physics of atmospheric gases, all enclosed or partially enclosed spaces experience infiltration. Whether it is through open doors and windows, or cracks and gaps in a building's walls, some amount of air will flow through seemingly air-tight building surfaces. Measuring this unintentional and uncontrolled entry of airflow provides a means to evaluate and analyze the conditions affecting indoor air quality and comfort. Rates of air infiltration have been clearly established to maintain proper indoor air quality and standards for public health. Rates of air infiltration also provide engineers with a means to evaluate a number of situations such as pipe freezes, children being trapped in hot cars, or the unwanted buildup of toxic or flammable gases.

A common method of evaluating air infiltration is completed by the utilization of a "tracer gas." In this process, a specific, unique gas molecule is released into an enclosed space of interest and then it's concentration is measured at fixed time intervals (typically 5 minutes). The air change rate (ACH) is calculated by the logarithmic value of the change of tracer gas concentration between two time intervals (less ambient levels) divided by the change in time.

Commonly, a specific type of tracer gas was selected so that there would be unique gas molecules that would not normally be found in the ambient air or within occupied building spaces. Since such gases would be rare, they could be easily distinguished from the air molecules found in the enclosed space. Thus, their concentrations could be easily distinguished and measured over time.

As mentioned previously, a tracer gas, when used in air infiltration studies, has typically been a gas that exists very rarely in nature. Historically, these may include, but not be limited to: acetone, helium, radioactive argon, radioactive krypton, methane, nitrous oxide, perfluorodimethylcyclohexane (PDCH), perfluoromethylcyclohexane (PMCH), or sulfur hexa-flouride.

It has been documented that CO_2 monitors/analyzers using ***nondispersive infrared technology*** (**NDIR**) are a fraction of the cost of monitors/analyzers for other types of tracer gas that use ***Fourier Transform Infrared spectroscopy*** (**FTIR**). In summary, **FTIR** gas monitoring equipment can be **50 - 100** times the cost of ***NDIR*** technology. **NDIR** sensors are commonly used for measuring CO_2 concentrations; high quality CO_2 monitors can have sensitivities of 20-50 PPM; typical **NDIR** CO_2 monitors are commonly $100 to $1,000 (U.S.). CO_2 can be utilized as a tracer gas as an alternative to other gases such as nitrous oxide (N_2O), sulfur hexafluoride (SF_6), or any of the perfluorocarbon tracer gases (PFTs). Although CO_2 may be used as a substitute for other tracer gases in many applications, there will continue to be applications where CO_2 would not be the best choice of tracer gas to use.

The use of CO_2 as a tracer gas has prompted the need to develop improved protocols for various scientific/engineering applications. The existing equations in ASTM E741 do not account for background levels of CO2 and are not compatible with any self-generation of tracer gas. Since CO_2 has a measurable presence in the ambient atmosphere (~400 ppm) and can be self-generated by the testing technician due to human respiration (breath exhalation) of CO_2, the methodology/mathematics for CO_2 tracer gas air infiltration testing was revised/corrected to provide improved accuracy.

The updates in methodology were necessary since other types of tracer gases have little if any ambient concentrations and are not self-generated by the testing technician. The use of discrete equations along with the use of numerical techniques enable the solution of infiltration rates even with CO_2 self-generation of 1-10 MET (1.0 MET = 0.00733 cfm of CO_2) and background levels of CO_2 with error rates of less than 5%. The use of discrete equations, and numerical techniques that are presented in this paper have been demonstrated to be accurate and can directly account for any internal generation of tracer gas as well as any ambient concentration of tracer gas.

This paper will highlight the scientific and engineering applications for tracer gas testing and provide detailed documentation of the methodology, mathematics, and testing protocols for CO_2 infiltration testing. The presented methodology has been brought forward to offer improved diagnostic opportunities for ***ASTM E741***. The reduced costs for using CO_2 monitors would also enable tracer gas testing for more applications and a greater number of engineers/scientists; the documentation of the analytic methods will serve to enable a more straight-forward completion of infiltration calculations and analysis.

The evaluation of indoor CO_2 concentrations as a determination of indoor air quality is based on CO_2 produced by human respiration. As it is a product of animal respiration, all aerobic organisms produce CO_2 when they oxidize carbohydrates.

Chemical Reaction Description of CO_2 Production

CO_2 is produced by each of the following processes:

a) A product of combustion for all fuels containing carbon (propane, gasoline, natural gas).

   The combustion of methane (CH_4):

   $$CH_4 + 2\ O_2 \rightarrow CO_2 + 2\ H_2O$$

b) The fermentation of sugar produces CO_2. Yeast metabolizes sugar to produce ethanol and CO_2:

   $$C_6H_{12}O_6 \rightarrow 2\ C_2H_5OH + 2\ CO_2$$

c) The evaluation of indoor CO_2 concentrations as a determination of indoor air quality is based on CO_2 produced by human respiration. As it is a product of animal respiration, all aerobic organisms produce CO_2 when they oxidize proteins, carbohydrates, and fatty acids:

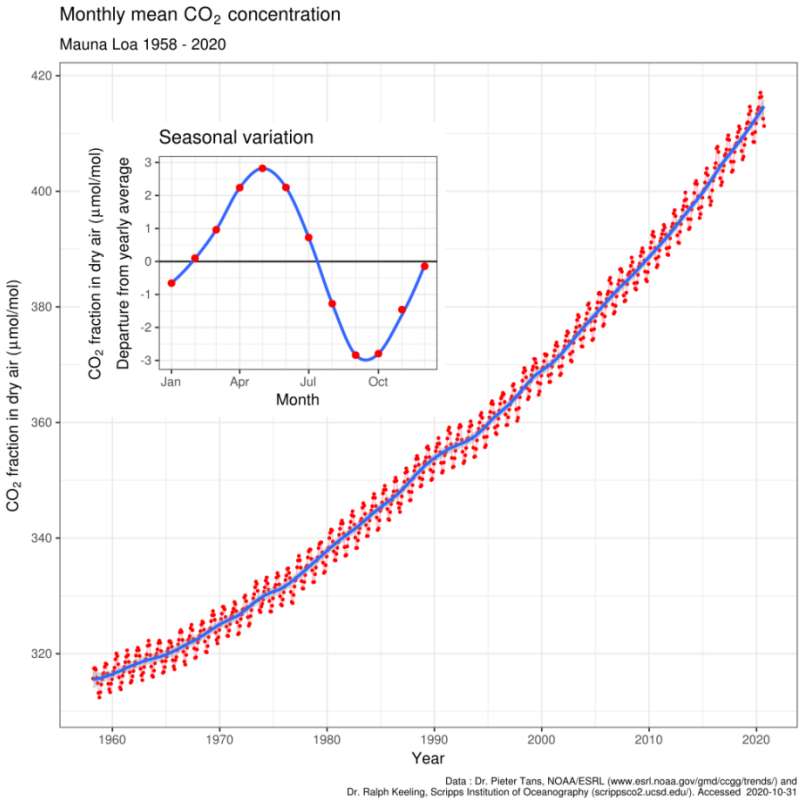
   $$C_6H_{12}O_6 + 6\ O_2 \rightarrow 6\ CO_2 + 6\ H_2O$$

Atmospheric Composition

Atmospheric air has primary components of nitrogen (N_2), oxygen (O_2), and argon (Ar) with a lesser proportion by volume of:

CO_2	Ne	He	CH_4
Kr	H_2	Xe	

The concentration of CO_2 has been increasing historically at approximately 0.5% per year. The present ambient atmospheric CO_2 concentrations are approximately 415 ppm.



Data from NOAA's Mauna Loa observatory showing the increase of carbon dioxide levels between the years 1958 – 2020.

figure 1

Historic CO_2 Concentration v. Year

Atmospheric Air Composition

CO_2 which has received much attention as a greenhouse gas and exists in approximately **0.04%** or 400 ppm (parts per million) concentration of the earth's atmosphere.

Atmospheric Air Gas Composition Distribution

		Molecular Weight [gm/mol]	Proportion (ppm)	Percent by Volume	Weight Percent	Molar Mass [gm/mol] air
Nitrogen	N_2	28.013	780,840	78.07878%	75.51130%	21.15298
Oxygen	O_2	31.999	209,460	20.94460%	23.13811%	7.40396
Argon	Ar	39.948	9,340	0.93394%	1.28805%	0.51455
Carbon dioxide	CO_2	44.01	400	0.04000%	0.06077%	0.02675
Neon	Ne	20.18	18	0.00182%	0.00127%	0.00026
Helium	He	4.003	5.2	0.00052%	0.00007%	0.00000
Methane	CH_4	16.042	1.8	0.00018%	0.00010%	0.00002
Krypton	Kr	83.798	1.0	0.00010%	0.00029%	0.00024
Hydrogen	H_2	2.016	0.5	0.00005%	0.00000%	0.00000
Xenon	Xe	131.293	0.1	0.00001%	0.00004%	0.00005
			CO_2 & Other:	0.04268%		0.02732
			Overall:	100.0%		29.12613

figure 2

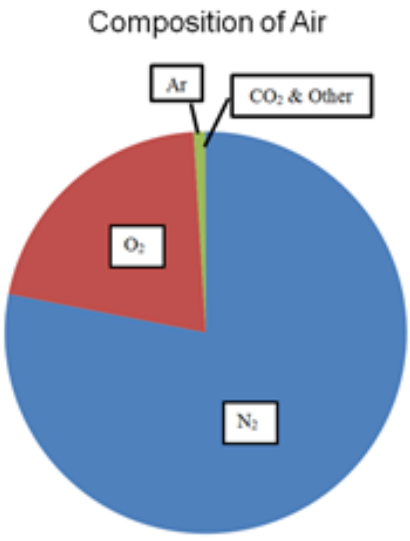


Figure 3 and Figure 4 confirm that carbon dioxide (CO_2) although measurable, exists as a relatively small portion of air (400 ppm or 0.04%) in comparison to nitrogen (N_2), oxygen (O_2), or argon (Ar).

ASTM E741 (Standard Test Method for Determining Air Change in a Single Zone by Means of a Tracer Gas Dilution), has been a well-established standard for the utilization of tracer gasses to determine air infiltration and has provided valuable information for my research.

Due to the need to determine air infiltration rates for various scientific/engineering applications and the high cost of other tracer gas instruments; CO_2 monitors are a fraction of the cost of monitors/analyzers for other types of tracer gas. CO_2 has been utilized as a tracer gas as an alternative to other gases such as nitrous oxide, sulfur hexafluoride, and methane.

The use of CO_2 as a tracer gas has prompted the need to develop improved protocols for various scientific/engineering applications. Since CO_2 has a measurable presence in the ambient (~400 ppm) and can be self-generated by the testing technician due to breath exhalation of CO_2, the methodology for CO_2 tracer gas air infiltration testing has been revised to provide reasonable accuracy, since other types of tracer gases have little if any ambient concentrations and are not self-generated by the testing technician or other building occupants.

The following methodology has been brought forward to advance science and improvements to ***ASTM E741 and ASTM D6245-18***.

Steady State Analysis

For steady state applications, with air infiltration and CO_2 generation rates constant, after a sufficient period of time, carbon dioxide concentrations will reach equilibrium.

It is noted that the quantitative volume of a room or space is not part of the analysis.

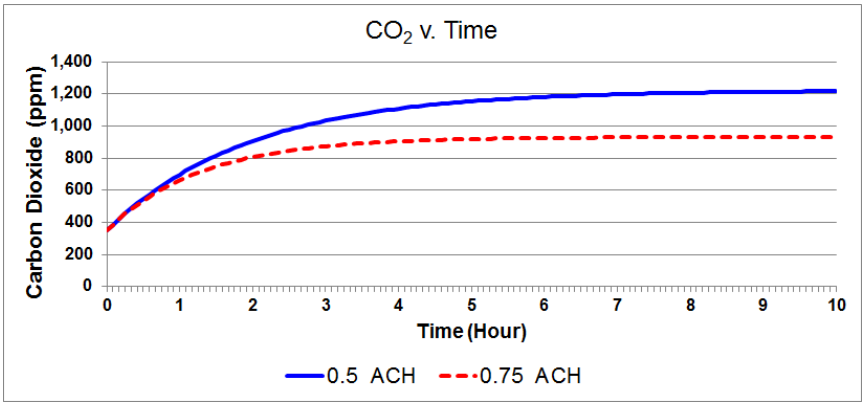
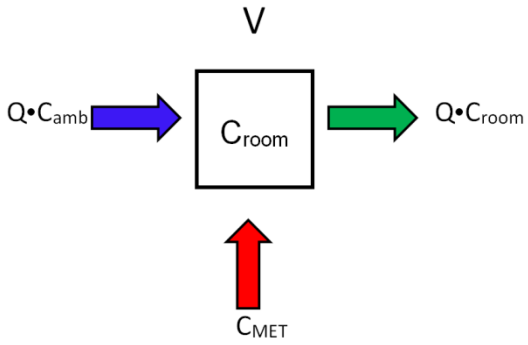


Figure 2

Figure 4

Carbon Balance



$\Sigma CO_2 = Q \bullet C_{amb} + C_{MET} = QC_{rc}$ (Eq. #A)

$C_{MET} = Q \bullet (C_{room} - C_{amb})$ (Eq. #B)

$Q = C_{MET}/(C_{room} - C_{amb})$ (Eq. #C)

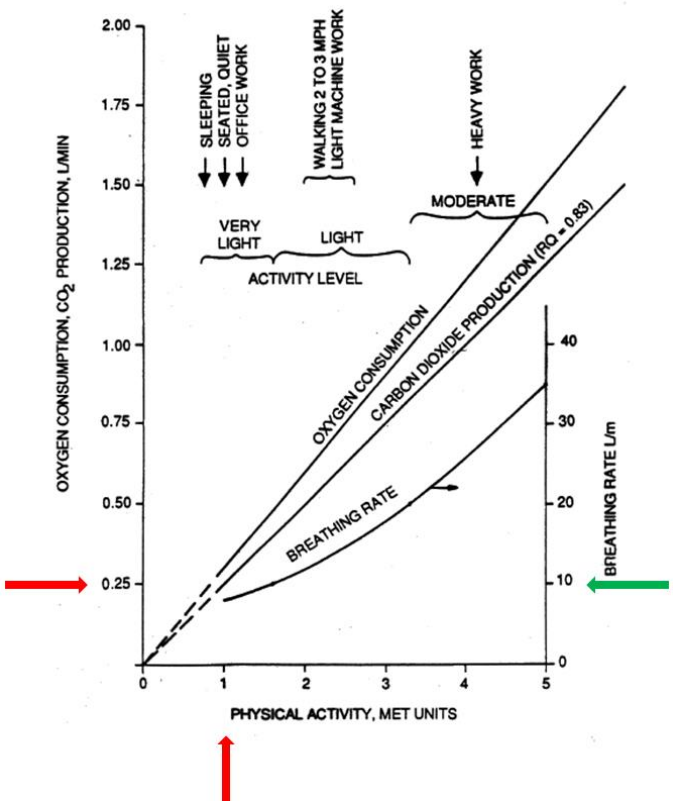
Q= Infiltration rate (volumetric air flow per time interval)

C_{MET} = G = CO_2 generation (volume of CO_2 per time interval)

CO_2 and Metabolic Equivalent Task (MET)

One **m**etabolic **e**quivalent **t**ask (**MET**) has been determined to be the amount of oxygen (O_2) consumed by an average human while sitting, in quiet study, or at prayer. For a **MET** activity of 1.0, O_2 levels have been documented to be 3.5 ml O_2 per kg body weight per minute. For a 70 kg person, O_2 consumption would be (70 kg x 3.5 ml/kg per minute) = 245 ml/min or 0.245 l/min. The respiratory quotient (RQ) is the volumetric ratio of CO_2 produced to oxygen (O_2) consumed.

RQ = CO_2/O_2 = 0.83 or 83%.



MET	Description
0.9	Sleeping
1	Quiet Study, Prayer
1-2	Driving Car
2-3	Golf
4-9	Tennis
7-8	Chopping Wood
10	Lap Swimming
16	6-minute mile

Note: 10 l/min = 0.35 cfm

Figure 6

Metabolic Data[1]

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Recent concerns regarding the COVID-19 virus present an excellent example in the use of carbon dioxide as an indicator of the risk and potential contamination concerns from this virus. In this situation, CO_2 as a human exhalation product can be linked to the human expiration of the virus. While there is no yet established mathematical calculation to directly calculate the expiration of CO_2 quantities to that of expired virus molecules, it can be safely assumed that higher CO_2 concentrations in a space can be associated with a potential for a higher virus concentration in that space as well. Therefore, by calculating CO_2 accumulation based on human activity in that space and then conducting an air infiltration study using CO_2, it is possible to assess the risk to occupants working or operating in that environment to the COVID-19 virus. Common best practices for indoor air quality recommend ventilation rates to maintain CO_2 concentrations of 800 - 1,000 ppm (please see chart below).

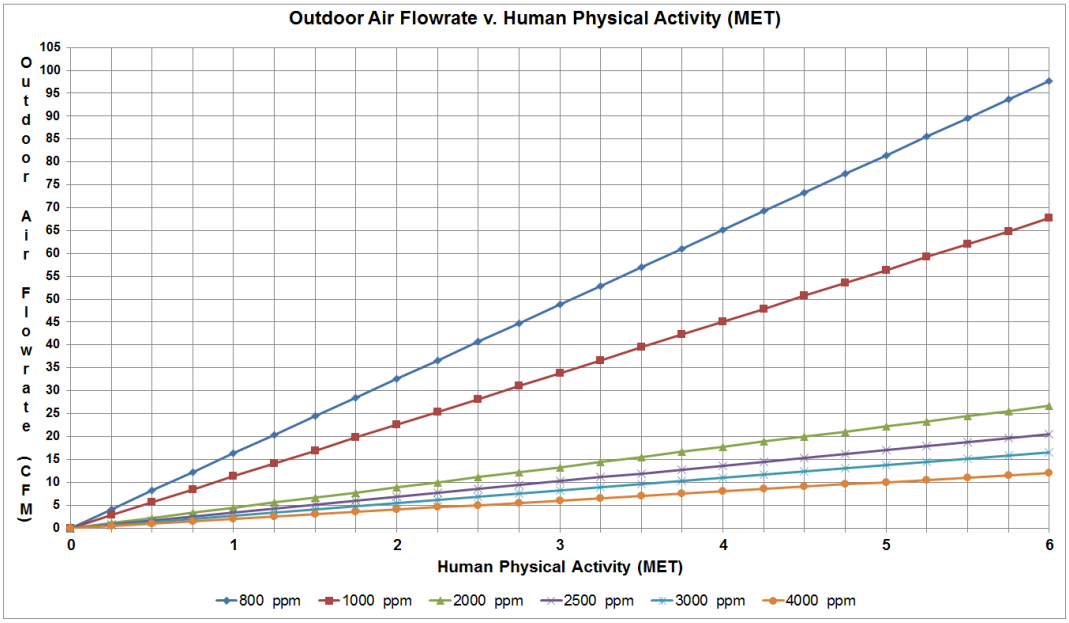


figure 7

Where:

$Q = C_{MET}/(C_{room} - C_{amb})$ (Eq. #C)

Q= Infiltration rate (volumetric air flow per time interval)

C_{MET} = G = CO_2 generation (volume of CO_2 per time interval)

C_{amb} = Ambient/Outdoor CO_2 concentration

C_{room} = Indoor (room) CO_2 concentration

Theory of Equilibrium Analysis (Conventional Analysis)

The mass balance of carbon dioxide is defined as follows:

$$V\frac{dC}{dt} = Q[C_{amb} - C_{(t)}] + G_{(t)} \quad (1)$$

V = Room volume

$\frac{dC}{dt}$ = Differential change of CO_2 concentration v. time

$C_{(t)}$ = Indoor CO_2 concentration at time t

Q = Infiltration volume per time interval

C_{amb} = Outdoor CO_2 concentration

$G_{(t)}$ = CO_2 generation flowrate at time t

Note: $G_{(t)}$ is constant; $G = G_{(t)}$

Then:

$$V\frac{dC}{dt} = Q[C_{amb} - C_{(t)}] + G \quad (2)$$

V = Room volume

$C_{(t)}$ = Indoor CO_2 concentration at time t

Q = Infiltration volume per time interval

C_{amb} = Ambient/Outdoor CO_2 concentration

G = Constant CO_2 generation volumetric flowrate

Note: Q, G, and C_o are constant

$$C_{(t)} = C_{amb} + G/Q + (C_{(i)} - C_{amb} - G/Q)e^{-It} \quad (3)$$

Note: The indoor concentration $C_{(t)}$ is a continuous exponential equation.

$$C(t) = C_{amb} + G/Q + (C(i) - C_{amb} - G/Q)e^{-(Q/V)t} \quad (3.a)$$

Q = Infiltration volume per time interval

V = Room volume

I = Q/V

$C_{(t)}$ = Indoor CO_2 concentration at time t

C_{amb} = Ambient/Outdoor CO_2 concentration

$C_{(i)}$ = Initial CO_2 concentration

G = CO_2 generation (volume of CO_2 per time interval)

Iterative Analysis Utilizing Discrete Equations

In order to model the change in concentration of carbon dioxide in real time, the volume of the room or space must be included in the analysis. The blending of air and carbon dioxide is documented by a carbon balance see **Figure 3** below:

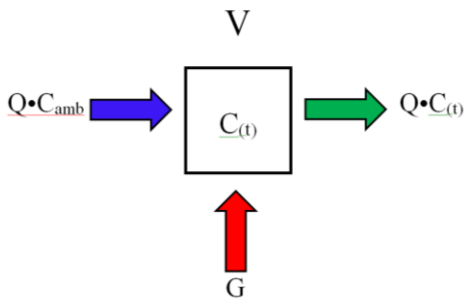


figure 8
Carbon Dioxide Balance

Note: Eq. (4) is equivalent/similar to Eq. (2)

$$\Sigma CO_2 = Q \bullet C_{amb} + G + V \bullet C_{(t)} - Q \bullet C_{(t)} \qquad (4)$$

CO_2 = Concentration: 1 ppm = cf of CO_2 per 1,000,000 cf of air
G – cf of CO_2 per time interval; i.e.: 1 MET = 0.00733 cf CO_2 per minute

1 MET = Metabolic human activity level for a typical 70 kg person

(t) = present interval iteration

Note: Eq. (5), (6), (7), (8), (9), & (10) provide an iterative solution for the discrete equation below:

$$C_{t+1} = \frac{V \cdot C_t + G + Q \cdot C_{amb} - Q \cdot C_t}{V} = \frac{\Sigma CO2}{V} \qquad (5)$$

CO_2 = Concentration: 1 ppm = m^3 of CO_2 per 1,000,000 m^3 of air
G – cf of CO_2 per time interval, i.e.: 1 MET = 0.00733 cf CO_2 per minute

1 MET = Metabolic human activity level for typical 70 kg person

(t) = present interval iteration; (t+1) = next interval iteration

$$C_{t+1} = C_t + G/V + \frac{Q \cdot [C_{amb} - C_t]}{V} = \frac{\Sigma CO2}{V} \qquad (5.a)$$

Note: Eq. (5) or Eq. (5a) can be iterated for any interval and time period using an electronic spreadsheet to determine a full range of values.

Solving for Q:

$$Q \cdot (C_{amb} - C_t) = C_{t+1} \cdot V - G - V \cdot C_t \qquad (6)$$

$$Q \cdot (C_t - C_{amb}) = V \cdot [C_t - C_{t+1}] + G$$

$$Q \text{ (per time interval)} = \frac{V \cdot [C_t - C_{t+1}] + G}{(C_t - C_{amb})} \qquad (7)$$

$$Q/V \text{ (per time interval)} = \frac{[C_t - C_{t+1}] + G/V}{(C_t - C_{amb})} \qquad (8)$$

Using Eq. (8), let ACH= Q/V:

$$\text{Air Change per Hour} = \text{ACH} = Q/V \bullet \frac{[60 \text{ minutes/hr}]}{[\text{Time Interval (Minutes)}]} \qquad (9)$$

Using Eq. (8), let ACH= Q/V:

$$\text{ACH} = \frac{[C_t - C_{t+1}] + G/V}{(C_t - C_{amb})} \bullet \frac{(60 \text{ minutes/hr})}{(\text{Interval Minutes})} \qquad (10)$$

Note: Eq. (9) and Eq. (10) are equivalent.

Perfect Gas Equations

The theory for tracer gas testing begins with traditional perfect gas equations.[10]

$PV = nR_uT$ *(11)*

P – Absolute pressure (lb/sf)

R_u=1545.32 (ft-lb)/[lb-mole ºR]

n=m/M; n – number of moles m = mass; M= molecular weight (lb/(lb-mole))

V = Volume (cf or ft^3)

For air @ 70°F:

M_{air} = 28.964 lb/(lb-mole)

$R_{air}=Ru/M_{air}$ = 1545.32 (ft-lb)/[lb-mole ºR] /28.964 [lb/(lb-mole)]

R_{air}= 53.353 ft/(ºR)

$PV = mR_{air}T$ *(12)*

$$\text{Density} = m/V = \frac{P}{(\text{RairT})} = \frac{(14.695\ \text{lb/si})(144\ \text{si/sf})}{(53.353\ \text{ft/ºR})(70ºF + 460ºF)} = 0.07483\ \text{lb/cf} \qquad (13)$$

For CO_2:

$m/V = P/(RCO_2T)$

M_{CO2} = 44.02 lb/(lb-mole)

R_{CO2} = Ru/M_{CO2} = 1545.32 (ft-lb)/[lb-mole ºR] /44.02 [lb/(lb-mole)]

R_{CO2}= 35.1 ft/(ºR)

Example, for CO_2 @ 70°F:

$$\text{Density} = m/V = \frac{P}{(\text{RCO2•T})} = \frac{(14.695\ \text{lb/si})(144\ \text{si/sf})}{(35.1\ \text{ft/ºR})(70ºF + 460ºF)} = 0.11374\ \text{lb/cf}$$

Example calculation using presented iterative analysis:

V = Room volume = 10 m • 10 m • 3m = 10,594 cf

Time interval = 5 minutes = 5/60 hr = 0.08333 hr

Q = Infiltration (cf per time interval)

C_{amb} = Ambient/Outdoor CO_2 concentration = 350 ppm = 0.000350

1.0 MET = 1.0 x 0.00733 cfm CO_2 for a typical person doing office work

G = [7 persons] • [1.5 MET • 0.00733 cfm CO_2/person] • [#min/time interval]

G = 7 • 1.5 • 0.00733 cfm CO_2/person x 5 minutes = 0.385 cf of CO_2

$C_{(t+1)}$ = 4753 ppm = 0.004753

$C_{(t)}$ = 5000 ppm = 0.00500

$$Q \text{ (per time interval)} == \frac{V \bullet [C_t - C_{t+1}] + G}{(C_t - C_{amb})} \qquad (7)$$

$$Q = \frac{10{,}594\,[0.005000 - 0.004753] + 0.385}{(0.005000 - 0.000350)}$$

Q = 644.39 cf in 5 minutes

Q = 7732.7 cf / hr

ACH = 7732.7/10594 = 0.73 (**97**% accuracy)

Note: For 5-minute intervals, accuracy improves to 99%

CO_2 Decay, Conventional Analysis (15 minute intervals)

$$C(t) = C_{amb} + G/Q + (C(i) - C_{amb} - G/Q)e^{-It} \qquad (3)$$

15-minute intervals
G = 7 people • 0.011 cfm CO_2/person • 15 minutes = 1.155 cf CO_2 per interval
V = Room volume = 10 m • 10 m • 3m = 10,594 cf
CO_2 ambient = 350 ppm

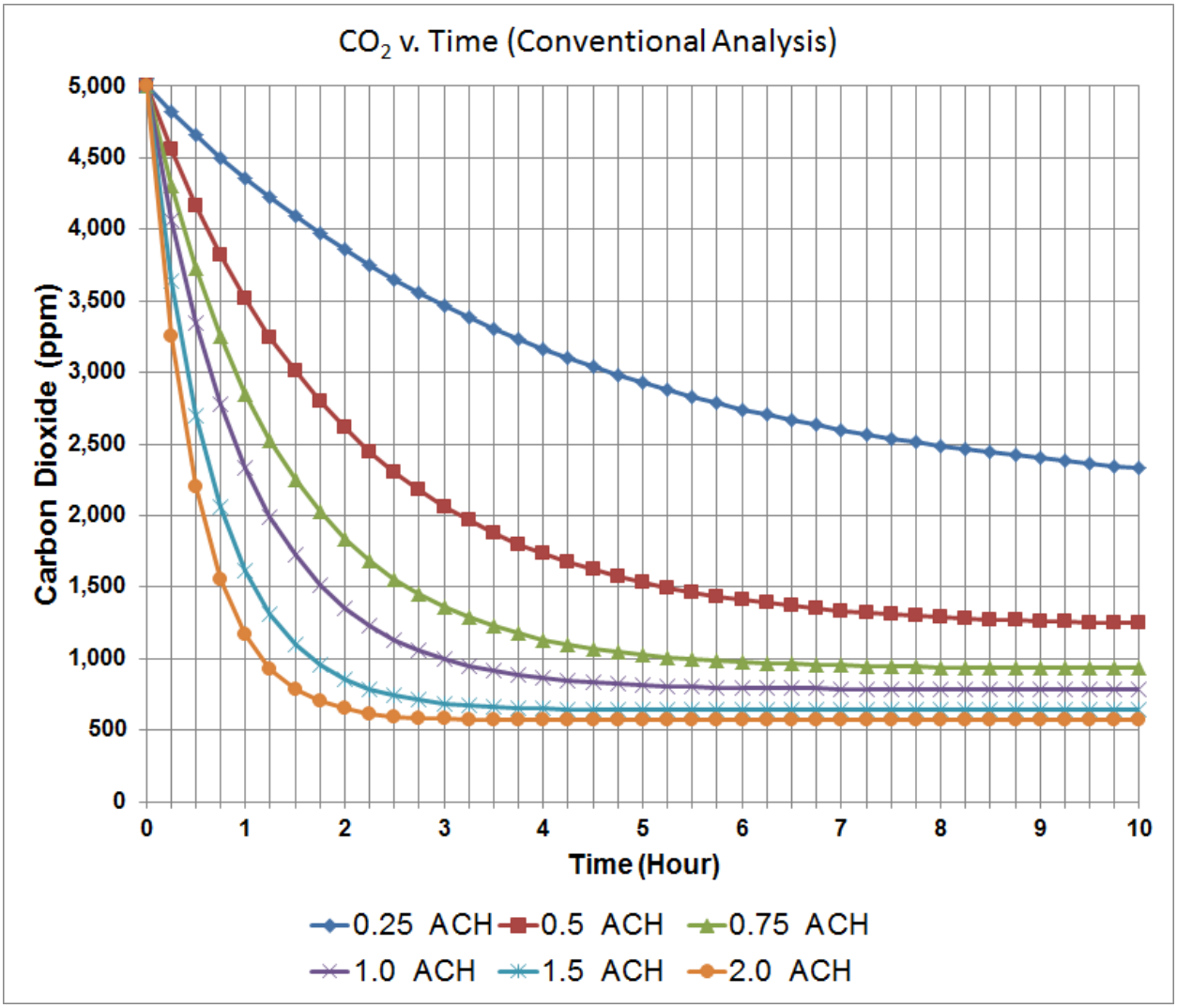


Figure 9

CO_2 Decay, Iterative Analysis (15-minute intervals):

$$C_{(t+1)} = \quad C_{t+1} = \frac{V \cdot C_t + G + Q \cdot C_{amb} - Q \cdot C_t}{V} = \sum \frac{CO2}{V} \qquad \text{(Eq. \#5)}$$

15-minute intervals

G = 7 people • 0.011 cfm CO_2/person • 15 minutes = 1.155 cf CO_2 per interval

V = Room volume = 10 m • 10 m • 3m = 10,594 cf

CO_2 ambient = 350 ppm

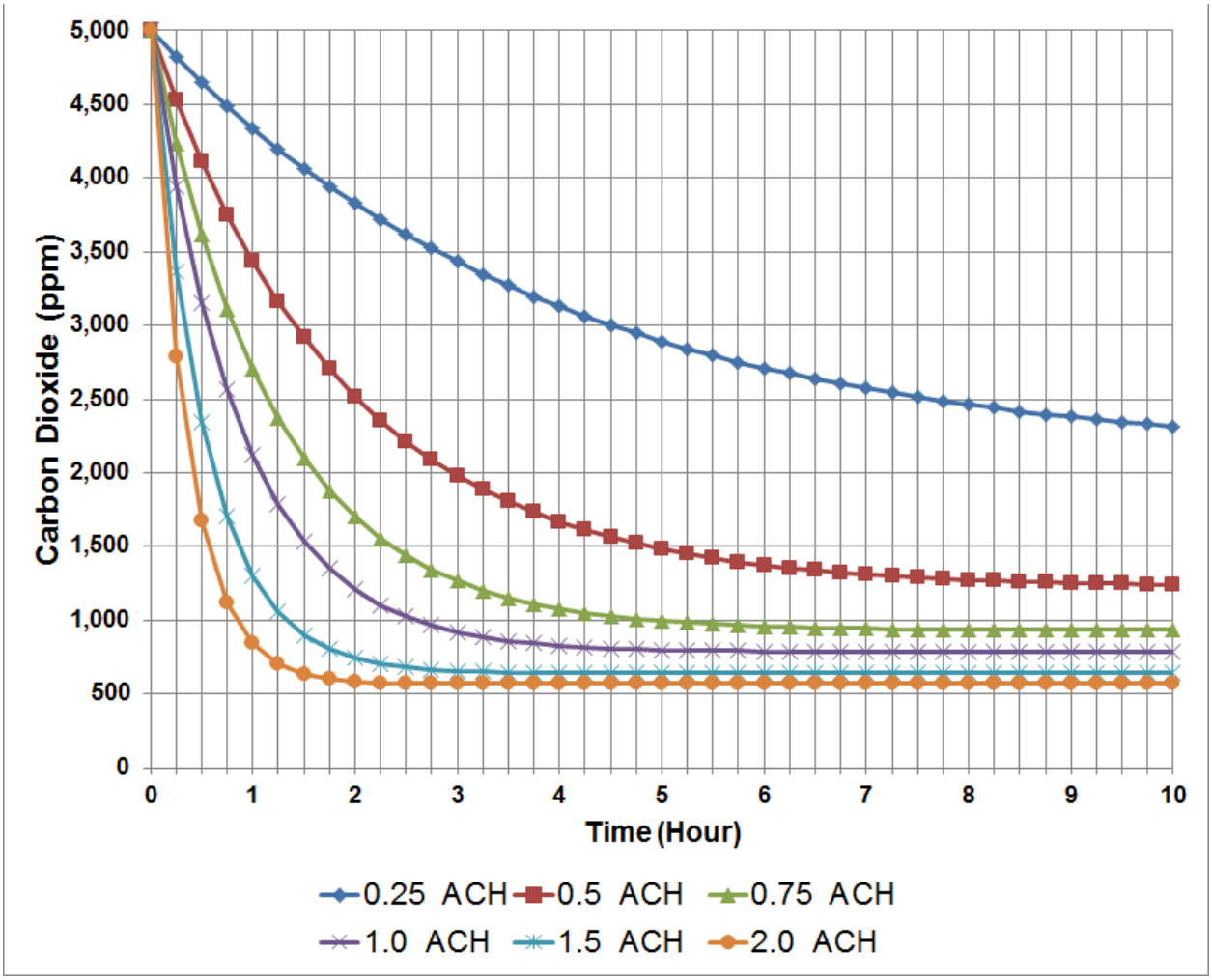


Figure 10

Comparison of Figure 9 and Figure 10

Conventional – **Black** (Equation #3)

Iterative Analysis – Blue (Equation #5)

	V	10,594		10,594		10,594		10,594		10,594		10,594	
	ACH	0.25	ACH	0.5	ACH	0.75	ACH	1.0	ACH	1.5	ACH	2.0	ACH
	Qinfil	662	cf/int	1,324		1,986		2,649		3,973		5,297	
		44.14	cfm										
Hr	G/Q:	0.0017444		0.0008722		0.0005815		0.0004361		0.0002907		0.0002180	
0	0	5000	5,000	5000	5,000	5000	5,000	5000	5,000	5000	5,000	5000	5,000
	0.25	4824	4,818	4556	4,528	4304	4,237	4068	3,947	3637	3,365	3256	2,784
	0.5	4659	4,648	4164	4,115	3728	3,617	3342	3,156	2700	2,344	2198	1,676
	0.75	4503	4,489	3819	3,753	3250	3,114	2777	2,564	2056	1,705	1557	1,122
1	1	4357	4,339	3514	3,437	2853	2,705	2336	2,119	1613	1,306	1168	845
	1.25	4220	4,199	3244	3,160	2525	2,372	1993	1,786	1309	1,056	932	707
	1.5	4091	4,067	3007	2,918	2252	2,102	1726	1,536	1100	901	789	637
	1.75	3970	3,944	2797	2,706	2026	1,883	1518	1,349	957	803	702	603
2	2	3857	3,828	2612	2,520	1839	1,704	1356	1,208	858	742	649	585
	2.25	3750	3,720	2449	2,358	1684	1,559	1230	1,102	790	704	617	577
	2.5	3650	3,618	2305	2,216	1555	1,442	1132	1,023	743	680	598	572
	2.75	3555	3,523	2177	2,092	1449	1,346	1055	964	711	666	586	570
3	3	3467	3,434	2065	1,983	1360	1,268	996	920	689	656	579	569
	3.25	3384	3,350	1966	1,888	1287	1,205	949	886	674	650	575	569
	3.5	3306	3,272	1879	1,805	1226	1,154	913	861	664	647	572	568
	3.75	3232	3,198	1802	1,732	1176	1,112	885	842	656	645	570	568
4	4	3163	3,129	1733	1,668	1134	1,078	863	828	652	643	570	568
	4.25	3099	3,064	1673	1,612	1099	1,051	846	818	648	642	569	568
	4.5	3038	3,004	1620	1,564	1071	1,028	833	810	646	642	569	568
	4.75	2981	2,947	1574	1,521	1047	1,010	823	804	644	641	568	568
5	5	2927	2,894	1532	1,484	1027	995	814	799	643	641	568	568
	5.25	2876	2,844	1496	1,451	1011	983	808	796	642	641	568	568
	5.5	2829	2,797	1464	1,422	997	974	803	794	642	641	568	568
	5.75	2785	2,753	1435	1,397	986	966	800	792	642	641	568	568
6	6	2743	2,712	1410	1,375	977	959	797	790	641	641	568	568
	6.25	2703	2,673	1388	1,356	969	954	794	789	641	641	568	568
	6.5	2667	2,637	1369	1,340	963	950	792	788	641	641	568	568
	6.75	2632	2,603	1351	1,325	957	946	791	788	641	641	568	568
7	7	2599	2,571	1336	1,312	953	944	790	787	641	641	568	568
	7.25	2569	2,541	1323	1,301	949	941	789	787	641	641	568	568
	7.5	2540	2,514	1311	1,291	946	939	788	787	641	641	568	568
	7.75	2513	2,487	1301	1,282	944	938	788	787	641	641	568	568
8	8	2488	2,463	1291	1,275	942	937	788	787	641	641	568	568
	8.25	2464	2,440	1283	1,268	940	936	787	786	641	641	568	568
	8.5	2441	2,418	1276	1,263	938	935	787	786	641	641	568	568
	8.75	2420	2,398	1270	1,257	937	934	787	786	641	641	568	568
9	9	2401	2,379	1264	1,253	936	934	787	786	641	641	568	568
	9.25	2382	2,361	1259	1,249	935	933	787	786	641	641	568	568
	9.5	2365	2,345	1255	1,246	935	933	786	786	641	641	568	568
	9.75	2348	2,329	1251	1,243	934	933	786	786	641	641	568	568
10	10	2333	2,314	1248	1,240	934	932	786	786	641	641	568	568

Iterative /Conventional:

Hr	ACH	0.25	0.50	0.75	1.00	1.50	2.00
0		1.000	1.000	1.000	1.000	1.000	1.000
		0.999	0.994	0.984	0.970	0.925	0.855
		0.998	0.988	0.970	0.944	0.868	0.762
		0.997	0.983	0.958	0.923	0.829	0.721
1		0.996	0.978	0.948	0.907	0.809	0.724
		0.995	0.974	0.940	0.896	0.807	0.758
		0.994	0.970	0.933	0.890	0.819	0.808
		0.993	0.967	0.929	0.888	0.840	0.859
2		0.993	0.965	0.927	0.891	0.865	0.902
		0.992	0.963	0.926	0.896	0.891	0.934
		0.991	0.962	0.927	0.904	0.915	0.957
		0.991	0.961	0.929	0.913	0.936	0.973
3		0.990	0.960	0.932	0.923	0.952	0.983
		0.990	0.960	0.936	0.933	0.965	0.989
		0.990	0.961	0.941	0.943	0.975	0.993
		0.989	0.961	0.946	0.952	0.982	0.996
4		0.989	0.962	0.951	0.960	0.987	0.998
		0.989	0.964	0.956	0.966	0.991	0.998
		0.989	0.965	0.960	0.972	0.994	0.999
		0.989	0.967	0.965	0.977	0.995	0.999
5		0.989	0.968	0.969	0.982	0.997	1.000
		0.989	0.970	0.973	0.985	0.998	1.000
		0.989	0.972	0.976	0.988	0.998	1.000
		0.989	0.974	0.980	0.990	0.999	1.000
6		0.989	0.975	0.982	0.992	0.999	1.000
		0.989	0.977	0.985	0.994	0.999	1.000
		0.989	0.979	0.987	0.995	1.000	1.000
		0.989	0.980	0.989	0.996	1.000	1.000
7		0.989	0.982	0.990	0.997	1.000	1.000
		0.989	0.983	0.992	0.997	1.000	1.000
		0.990	0.985	0.993	0.998	1.000	1.000
		0.990	0.986	0.994	0.998	1.000	1.000
8		0.990	0.987	0.995	0.999	1.000	1.000
		0.990	0.988	0.996	0.999	1.000	1.000
		0.990	0.989	0.996	0.999	1.000	1.000
		0.991	0.990	0.997	0.999	1.000	1.000
9		0.991	0.991	0.997	1.000	1.000	1.000
		0.991	0.992	0.998	1.000	1.000	1.000
		0.991	0.993	0.998	1.000	1.000	1.000
		0.992	0.993	0.998	1.000	1.000	1.000
10		0.992	0.994	0.999	1.000	1.000	1.000
	Average:	0.991	0.977	0.969	0.965	0.959	0.956
						Overall:	0.970
						Error:	-3.03%

For 5-minute Intervals:

Average:	0.997		0.992		0.990		0.988		0.986		0.985
										Overall:	0.990
										Error:	-1.03%

CO_2 Growth, Conventional Analysis (15-minute intervals):

$$C(t) = C_{amb} + G/Q + (C(i) - C_{amb} - G/Q)e^{-It} \quad \text{(Eq. \#3)}$$

15-minute intervals

G = 7 people • 0.011 cfm CO_2/person • 15 minutes = 1.155 cf CO_2 per interval

V = Room volume = 10 m • 10 m • 3m = 10,594 cf

CO_2 ambient = 350 ppm

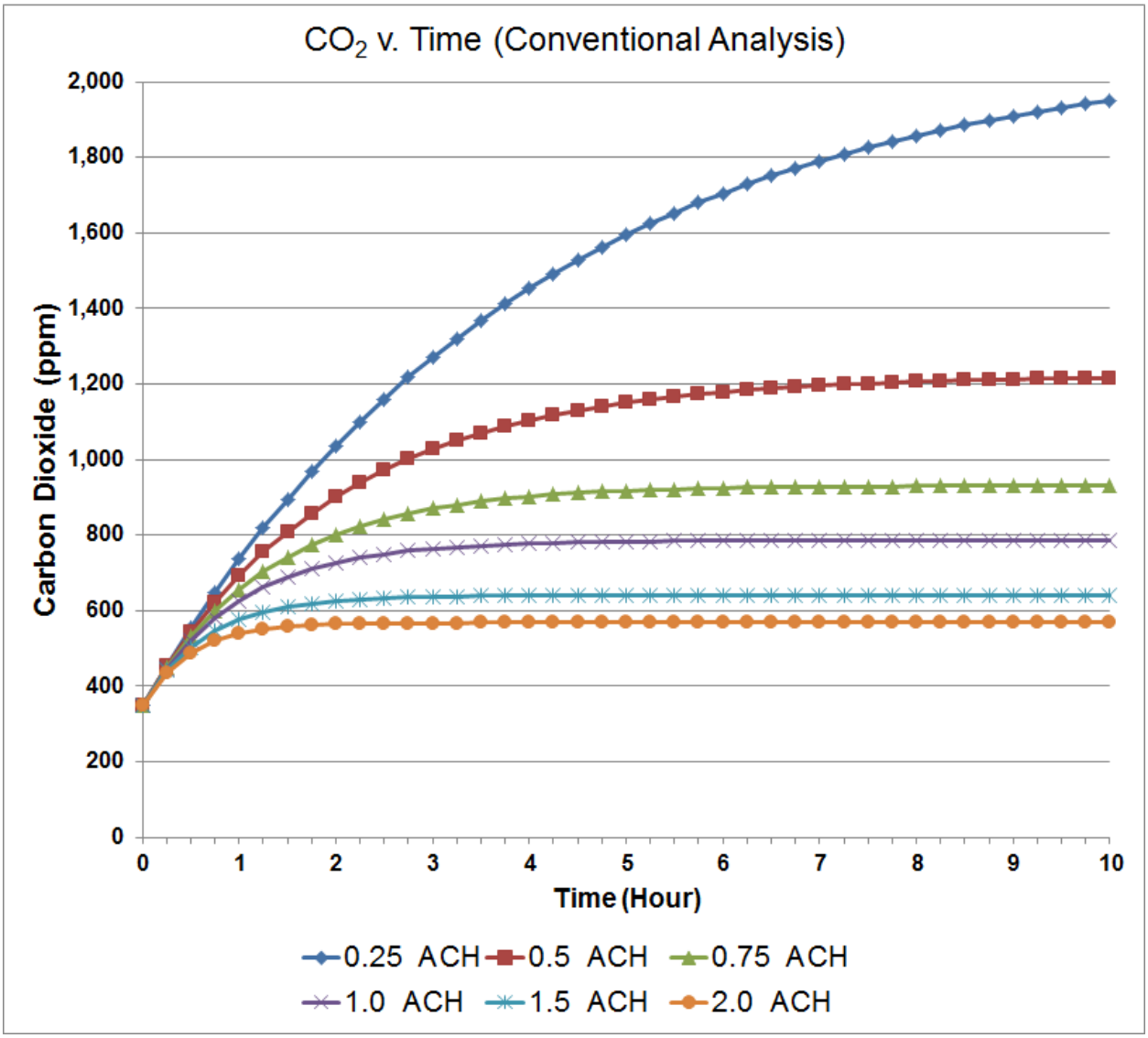
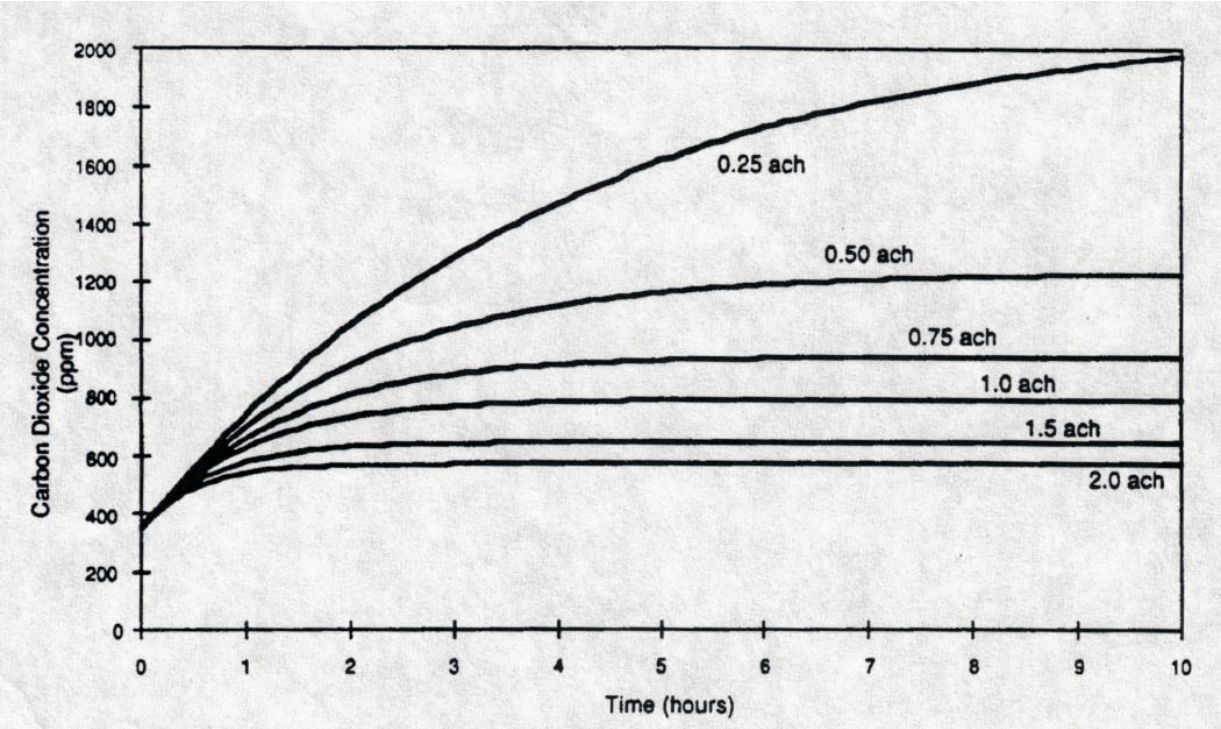


Figure 11

"***A Study of Ventilation and Carbon Dioxide in an Office Building***,"
Nabinger, Persily, & Dols, 1994:



CO2 V. Time for ACH of 0.25 – 2.0

Figure 12

Note: The above article reported that Figure 7 above utilized the following parameters:

G = 7 people • 0.011 cfm CO_2/person = 0.077 cfm CO_2
V = Room volume = 10 m • 10 m • 3m = 300 m^3 = 10,594 cf
CO_2 ambient = 350 ppm

Figure 7 was produced applying Eq. #3:

$$\mathbf{C(t) = C_{amb} + G/Q + (C(i) - C_{amb} - G/Q)e^{-It}} \qquad \text{(Eq. \#3)}$$

The above values for "G" and "V" were the same values used in Figure 6; Figure 6 and Figure 7 are therefore the same/similar.

CO_2 Growth, Iterative Analysis (15-minute intervals):

$$C_{(t+1)} = \frac{V \cdot C_t + G + Q \cdot C_{amb} - Q \cdot C_t}{V} = \sum \frac{CO2}{V} \qquad \text{(Eq. \#5)}$$

15-minute intervals

G = 7 people • 0.011 cfm CO_2/person • 15 minutes = 1.155 cf CO_2 per interval

V = Room volume = 10 m • 10 m • 3m = 10,594 cf

CO_2 ambient = 350 ppm

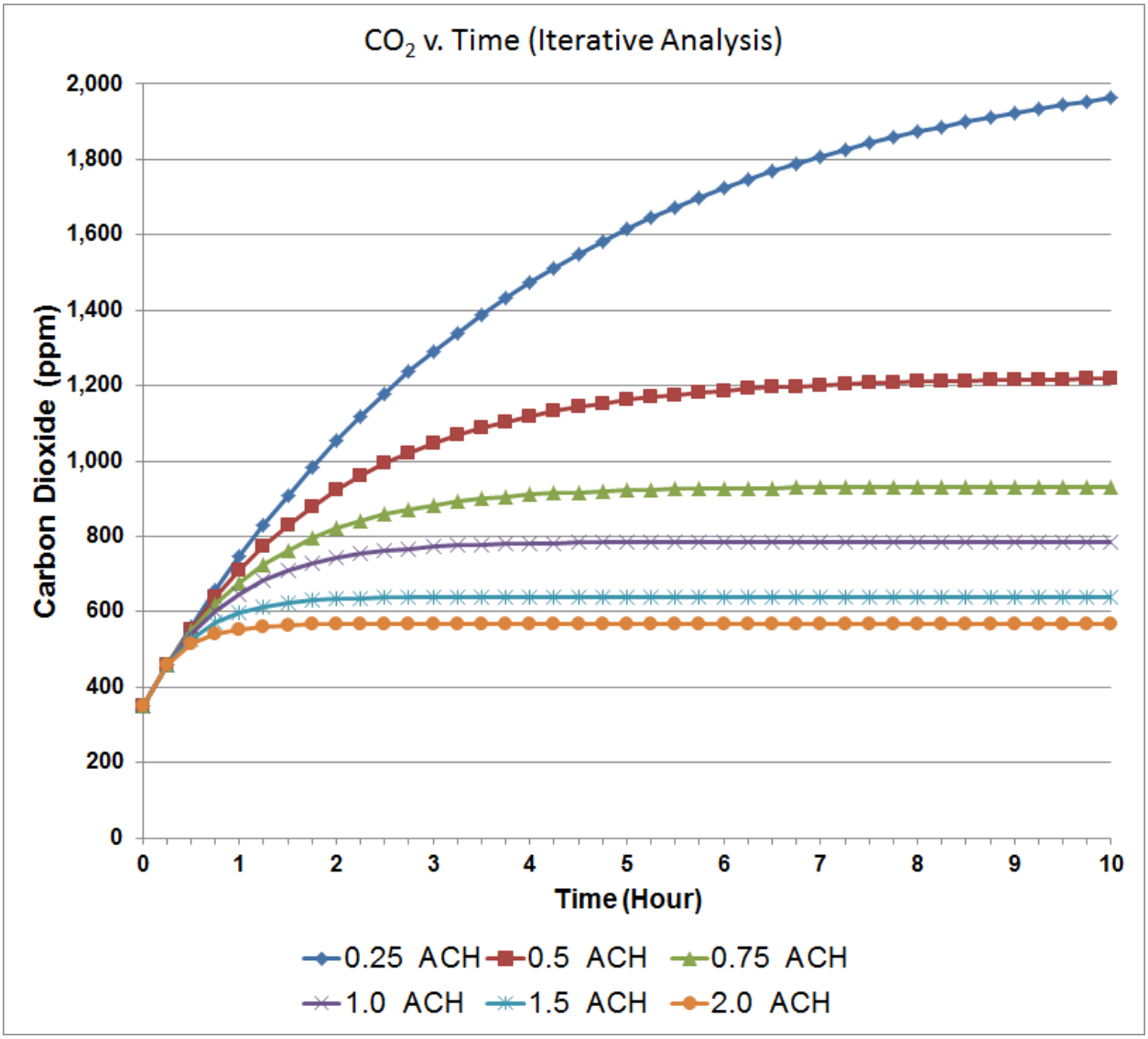


Figure 13

Comparison of Figure 11 and Figure 13

Conventional – **Black** (Equation #3)

Iterative Analysis – Blue (Equation #5)

	V	10,594		10,594		10,594		10,594		10,594		10,594	
	ACH	0.25	ACH	0.5	ACH	0.75	ACH	1.0	ACH	1.5	ACH	2.0	ACH
	Qinfil	662	cf/int	1,324		1,986		2,649		3,973		5,297	
		44.14	cfm										
Hr	G/Q:	0.0017444		0.0008722		0.0005815		0.0004361		0.0002907		0.0002180	
0	0	350	350	350	350	350	350	350	350	350	350	350	350
	0.25	456	459	452	459	449	459	446	459	441	459	436	459
	0.5	555	561	543	554	532	548	522	541	503	527	488	514
	0.75	648	657	623	638	600	620	580	602	546	570	519	541
1	1	736	747	693	711	657	678	626	648	576	596	539	554
	1.25	818	831	755	775	704	726	661	683	596	613	550	561
	1.5	895	910	810	831	743	764	689	708	610	623	557	565
	1.75	968	984	859	880	775	796	710	728	620	630	561	566
2	2	1036	1,053	901	922	802	821	727	742	626	634	564	567
	2.25	1100	1,119	939	960	824	842	740	753	631	636	566	568
	2.5	1161	1,180	972	993	842	859	750	762	634	638	567	568
	2.75	1217	1,237	1002	1,021	858	872	758	768	636	639	567	568
3	3	1270	1,290	1028	1,047	870	883	764	772	638	640	568	568
	3.25	1320	1,341	1050	1,068	881	892	769	776	639	640	568	568
	3.5	1367	1,388	1071	1,088	889	900	773	778	639	640	568	568
	3.75	1411	1,432	1088	1,105	897	906	776	780	640	640	568	568
4	4	1453	1,473	1104	1,119	903	910	778	782	640	641	568	568
	4.25	1492	1,512	1118	1,132	907	914	780	783	640	641	568	568
	4.5	1528	1,548	1130	1,143	912	918	781	784	640	641	568	568
	4.75	1562	1,583	1141	1,153	915	920	782	784	640	641	568	568
5	5	1595	1,615	1151	1,162	918	922	783	785	641	641	568	568
	5.25	1625	1,645	1159	1,169	920	924	784	785	641	641	568	568
	5.5	1653	1,673	1166	1,176	922	925	784	785	641	641	568	568
	5.75	1680	1,699	1173	1,182	924	927	785	786	641	641	568	568
6	6	1705	1,724	1179	1,187	925	927	785	786	641	641	568	568
	6.25	1729	1,747	1184	1,191	926	928	785	786	641	641	568	568
	6.5	1751	1,769	1188	1,195	927	929	785	786	641	641	568	568
	6.75	1772	1,789	1192	1,198	928	929	786	786	641	641	568	568
7	7	1791	1,808	1196	1,201	928	930	786	786	641	641	568	568
	7.25	1810	1,826	1199	1,204	929	930	786	786	641	641	568	568
	7.5	1827	1,843	1202	1,206	929	930	786	786	641	641	568	568
	7.75	1843	1,858	1204	1,208	930	931	786	786	641	641	568	568
8	8	1858	1,873	1206	1,210	930	931	786	786	641	641	568	568
	8.25	1873	1,887	1208	1,212	930	931	786	786	641	641	568	568
	8.5	1886	1,900	1210	1,213	930	931	786	786	641	641	568	568
	8.75	1899	1,912	1211	1,214	931	931	786	786	641	641	568	568
9	9	1911	1,924	1213	1,215	931	931	786	786	641	641	568	568
	9.25	1922	1,934	1214	1,216	931	931	786	786	641	641	568	568
	9.5	1932	1,944	1215	1,217	931	931	786	786	641	641	568	568
	9.75	1942	1,954	1216	1,217	931	931	786	786	641	641	568	568
10	10	1951	1,962	1216	1,218	931	931	786	786	641	641	568	568

Iterative /Conventional:

Hr	ACH	0.25	0.50	0.75	1.00	1.50	2.00
0		1.000	1.000	1.000	1.000	1.000	1.000
		1.007	1.014	1.021	1.028	1.041	1.053
		1.011	1.021	1.030	1.037	1.047	1.053
		1.014	1.024	1.032	1.038	1.043	1.041
1		1.015	1.026	1.032	1.036	1.036	1.029
		1.016	1.026	1.031	1.032	1.028	1.020
		1.016	1.025	1.029	1.029	1.022	1.013
		1.016	1.025	1.027	1.025	1.017	1.009
2		1.017	1.023	1.024	1.021	1.012	1.006
		1.016	1.022	1.022	1.018	1.009	1.004
		1.016	1.021	1.019	1.015	1.007	1.002
		1.016	1.020	1.017	1.012	1.005	1.001
3		1.016	1.018	1.015	1.010	1.003	1.001
		1.015	1.017	1.013	1.009	1.002	1.001
		1.015	1.016	1.012	1.007	1.002	1.000
		1.015	1.015	1.010	1.006	1.001	1.000
4		1.014	1.014	1.009	1.005	1.001	1.000
		1.014	1.013	1.008	1.004	1.001	1.000
		1.013	1.012	1.007	1.003	1.000	1.000
		1.013	1.011	1.006	1.002	1.000	1.000
5		1.013	1.010	1.005	1.002	1.000	1.000
		1.012	1.009	1.004	1.002	1.000	1.000
		1.012	1.008	1.004	1.001	1.000	1.000
		1.011	1.007	1.003	1.001	1.000	1.000
6		1.011	1.007	1.003	1.001	1.000	1.000
		1.011	1.006	1.002	1.001	1.000	1.000
		1.010	1.006	1.002	1.001	1.000	1.000
		1.010	1.005	1.002	1.000	1.000	1.000
7		1.009	1.005	1.001	1.000	1.000	1.000
		1.009	1.004	1.001	1.000	1.000	1.000
		1.009	1.004	1.001	1.000	1.000	1.000
		1.008	1.003	1.001	1.000	1.000	1.000
8		1.008	1.003	1.001	1.000	1.000	1.000
		1.008	1.003	1.001	1.000	1.000	1.000
		1.007	1.003	1.001	1.000	1.000	1.000
		1.007	1.002	1.000	1.000	1.000	1.000
9		1.007	1.002	1.000	1.000	1.000	1.000
		1.007	1.002	1.000	1.000	1.000	1.000
		1.006	1.002	1.000	1.000	1.000	1.000
		1.006	1.002	1.000	1.000	1.000	1.000
10		1.006	1.001	1.000	1.000	1.000	1.000
	Average:	1.011	1.011	1.010	1.008	1.007	1.006
						Overall:	1.009
						Error:	0.88%

For 5-minute Intervals:

Average:	1.004		1.004		1.003		1.003		1.002		1.002
										Overall:	1.003
										Error:	0.29%

Example: Carbon Monoxide (**CO**) Balance; see figure below.

Note: Nomenclature has been modified to avoid confusion of "**CO**" with "**Co**"

Note: Eq. #11 is equivalent to Eq. #4

$$\Sigma CO = Q \bullet C_{amb} + CO_{Source} + V \bullet CO_{(t)} - Q \bullet CO_{(t)} \qquad \text{(Eq. \#11)}$$

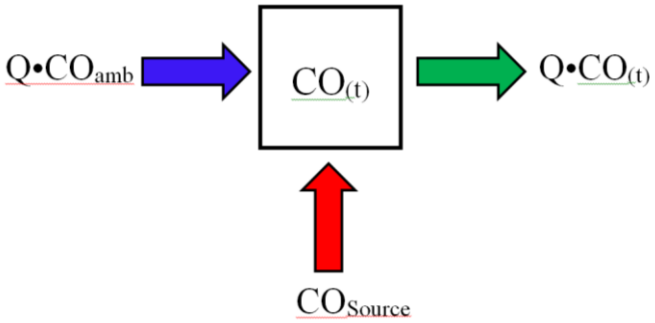


Figure 14
Carbon Monoxide Balance

CO Equation Parameters:

- Q ~ Volumetric air flow per time interval (cf/{Time interval})
- CO_{amb} – Ambient CO levels = 0 ppm
- $CO_{(t)}$ Concentration: 1 ppm = 1.0 cf of CO per 1,000,000 cf of air
- CO_{Source} – Source cf of CO per time interval;
- V – Room volume (cf)

Note: Eq. #12 is CO applied to Eq. #5

$$CO_{t+1} = \frac{V \cdot CO_t + CO_{Source} + Q \cdot CO_{amb} - Q \cdot CO_t}{V} = \frac{\Sigma CO}{V} \qquad \text{(Eq. \#12)}$$

Carbon Monoxide (CO) Sample Calculation Example (see Figure 15 below):

2" PVC (2.047" ID) vent discharge @ 1,200 fpm = 27.4 cfm

CO measured @ 1,000 ppm (@110°F) [Note: Room temperature = 70°F]

70°F Volume correction factor: (70°F+460)/(110°F +460°F) = 0.9298

Interval = 15 minutes

CO_{Source} = 27.4 cfm • 15 min/int • 1,000 ppm/1,000,000 • 0.9298 = 0.3825 cf of CO/int

V = 2000 cf • 8ft = 16,000 cf

Infiltration = 70% x V = 11,200 cf/h or 186.7 cf/min

After the first 15 minute interval:

- $CO_{t+1} = \frac{V \cdot CO_t + CO_{Source} + Q \cdot CO_{amb} - Q \cdot CO_t}{V}$
- $CO_{t+1} = \frac{16{,}000 \bullet 0 + 0.382 + 0 - 186.7 x 0}{16{,}000}$ = 0.00002391 =23.91 ppm =0.00002391

Next 15 minute interval:

- $CO_{t+1} = \frac{16{,}000 \bullet 0.00002391 + 0.3825 - 186.7 \bullet 0.00002391}{16{,}000}$ = 47.53 ppm

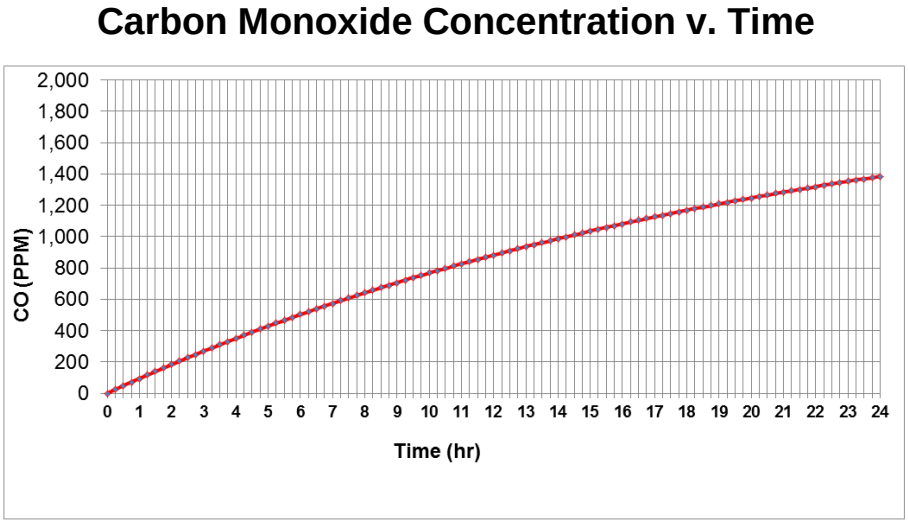


Figure 15

Calculation of "***Air Change Rate***" based on ASTM E741-11, section 8.5.3

8.5.3.1 Averaged Method – Calculate C_1, the average of the concentrations at the time t_1 that confirmed the uniformity of concentration criterion(8.4). Calculate C_2. The average of the concentrations at the time t_2 that confirmed the uniformity of concentration criterion at the end of the test (8.4.1). Calculate mean air change rate $\bar{A}$ by Equation (ii):

$$\bar{A} = [\ \ln C(t_1) - \ln C(t_2)\ ]/(t_2 - t_1) \qquad \text{Equation (ii)}$$

Note: "Equation (ii)" should not be confused with "Eq. #2."

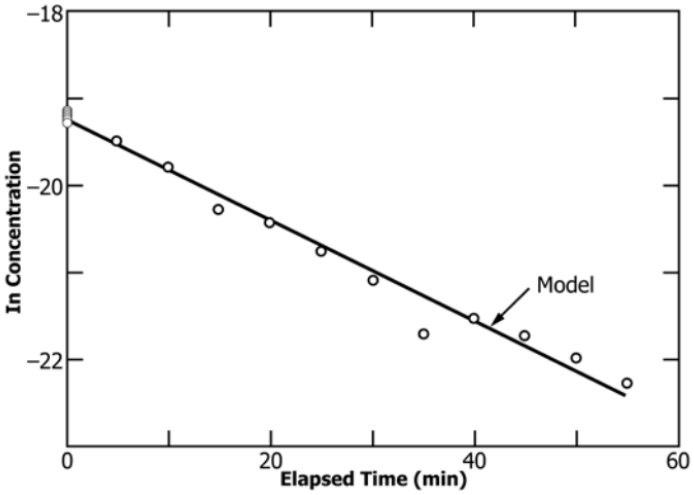


FIG. 2 Concentrations and Estimate of Concentrations Plotted Logarithmically

Figure 16

(Figure 2) from ASTM E741-11 (2017):

Note: The above **Figure 16** has been reprinted, with permission, from E741-11(2017) Standard Test Method for Determining Air Change in a Single Zone by Means of a Tracer Gas Dilution, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428. A copy of the complete standard may be obtained from ASTM International, www.astm.org.

The sensitivity analysis below will demonstrate that this method does not work if any of the following are true:

a) There is internal generation of the tracer gas;

b) There are significant ambient background levels of tracer gas;

Equation (ii) has been modified as follows to allow for background levels of tracer gas.

$\bar{A} = [\ \ln[C(t_1) - C_{amb}] - \ln[C(t_2) - C_{amb}]]/(t_2 - t_1)$ Equation (ii) modified

$C(t) = C_{amb} + e^{-I \cdot t}$ (Eq. #13)

$C(t) = C_{amb} + e^{-(Q/V) \cdot t}$ (Eq. #13a)

Equation Parameters:

- Q = Infiltration volume per time interval
- V = Room volume
- I = Q/V
- C(t) = Indoor CO_2 concentration at time t
- C_{amb} = Ambient/Outdoor CO_2 concentration

Evaluation of data, analysis example:
Using Eq. #3 below (conventional analysis), with a CO_2 generation rate of 7 persons @ 0.011 cf of CO_2 per minute.

$$C(t) = C_{amb} + G/Q + (C_{(i)} - C_{amb} - G/Q)e^{-It} \qquad \text{(Eq. \#3)}$$

Where:

Q = Infiltration volume per time interval
V = Room volume
I = Q/V
C(t) = Indoor CO_2 concentration at time t
C_{amb} = Ambient/Outdoor CO_2 concentration
$C_{(i)}$ = Initial CO_2 concentration
G = CO_2 generation (volume of CO_2 per time interval)
15-minute intervals
G = 7 people x 0.011 cfm CO_2/person x 15 minutes = 1.155 cf CO_2 per interval
V = Room volume = 10 m • 10 m • 3m = 10,594 cf
CO_2 ambient = 350 ppm

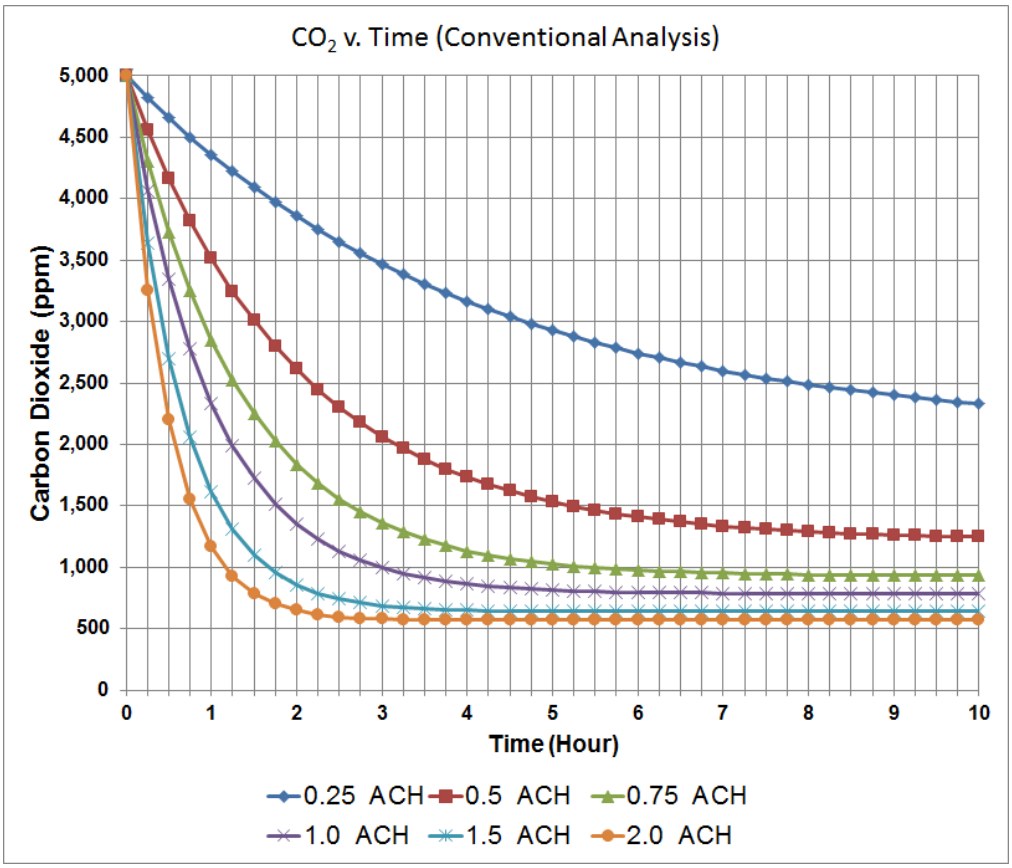


Figure 17

$$\bar{A} = [\ \ln C(t_2) - \ln C(t_1)\ ]/(t_2 - t_1) \qquad \text{Equation (ii)}$$

$\bar{A}$ was then compared to I or the true ACH found in Eq. #3.

$$C_{(t)} = C_{amb} + G/Q + (C_{(i)} - C_{amb} - G/Q)e^{-It} \qquad \text{(Eq. \#3)}$$

The determination of $\bar{A}$ and its error analysis are as follows:

Time	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH
(hr)	0.25	0.25	0.50	0.50	0.75	0.75	1.00	1.00	1.50	1.50	2.00	2.00
	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error
0	0.14	43%	0.37	26%	0.60	20%	0.83	17%	1.27	15%	1.72	14%
	0.14	44%	0.36	28%	0.58	23%	0.79	21%	1.19	21%	1.57	21%
	0.14	46%	0.35	31%	0.55	27%	0.74	26%	1.09	27%	1.38	31%
	0.13	47%	0.33	33%	0.52	31%	0.69	31%	0.97	35%	1.15	42%
1	0.13	49%	0.32	36%	0.49	35%	0.63	37%	0.84	44%	0.90	55%
	0.12	50%	0.30	39%	0.46	39%	0.58	42%	0.70	54%	0.67	67%
	0.12	52%	0.29	42%	0.42	44%	0.51	49%	0.56	63%	0.47	77%
	0.12	54%	0.27	45%	0.39	48%	0.45	55%	0.44	71%	0.31	84%
2	0.11	55%	0.26	48%	0.35	53%	0.39	61%	0.33	78%	0.20	90%
	0.11	57%	0.24	51%	0.32	58%	0.33	67%	0.24	84%	0.13	94%
	0.10	58%	0.23	55%	0.28	62%	0.28	72%	0.18	88%	0.08	96%
	0.10	60%	0.21	58%	0.25	66%	0.23	77%	0.13	92%	0.05	98%
3	0.10	61%	0.20	61%	0.22	70%	0.19	81%	0.09	94%	0.03	99%
	0.09	63%	0.18	64%	0.19	74%	0.16	84%	0.06	96%	0.02	99%
	0.09	64%	0.17	66%	0.17	78%	0.13	87%	0.04	97%	0.01	99%
	0.09	66%	0.15	69%	0.14	81%	0.10	90%	0.03	98%	0.01	100%
4	0.08	67%	0.14	72%	0.12	83%	0.08	92%	0.02	99%	0.00	100%
	0.08	68%	0.13	74%	0.11	86%	0.06	94%	0.01	99%	0.00	100%
	0.08	70%	0.12	77%	0.09	88%	0.05	95%	0.01	99%	0.00	100%
	0.07	71%	0.11	79%	0.08	90%	0.04	96%	0.01	100%	0.00	100%
5	0.07	72%	0.10	81%	0.06	91%	0.03	97%	0.00	100%	0.00	100%
	0.07	73%	0.09	83%	0.05	93%	0.02	98%	0.00	100%	0.00	100%
	0.06	75%	0.08	84%	0.05	94%	0.02	98%	0.00	100%	0.00	100%
	0.06	76%	0.07	86%	0.04	95%	0.01	99%	0.00	100%	0.00	100%
6	0.06	77%	0.06	87%	0.03	96%	0.01	99%	0.00	100%	0.00	100%
	0.05	78%	0.06	89%	0.03	96%	0.01	99%	0.00	100%	0.00	100%
	0.05	79%	0.05	90%	0.02	97%	0.01	99%	0.00	100%	0.00	100%
	0.05	80%	0.05	91%	0.02	98%	0.01	99%	0.00	100%	0.00	100%
7	0.05	81%	0.04	92%	0.02	98%	0.00	100%	0.00	100%	0.00	100%
	0.05	82%	0.04	93%	0.01	98%	0.00	100%	0.00	100%	0.00	100%
	0.04	83%	0.03	94%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.04	84%	0.03	94%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
8	0.04	85%	0.03	95%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.04	85%	0.02	96%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.03	86%	0.02	96%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.03	87%	0.02	96%	0.00	99%	0.00	100%	0.00	100%	0.00	100%
9	0.03	88%	0.02	97%	0.00	100%	0.00	100%	0.00	100%	0.00	100%
	0.03	88%	0.01	97%	0.00	100%	0.00	100%	0.00	100%	0.00	100%
	0.03	89%	0.01	98%	0.00	100%	0.00	100%	0.00	100%	0.00	100%
	0.03	89%	0.01	98%	0.00	100%	0.00	100%	0.00	100%	0.00	100%
10	0.02	90%	0.01	98%	0.00	100%	0.00	100%	0.00	100%	0.00	100%

15-minute intervals

G = **0** Cf CO_2 per interval

V = Room volume = 10 m • 10 m • 3m = 10,594 cf

CO_2 ambient = **350** ppm

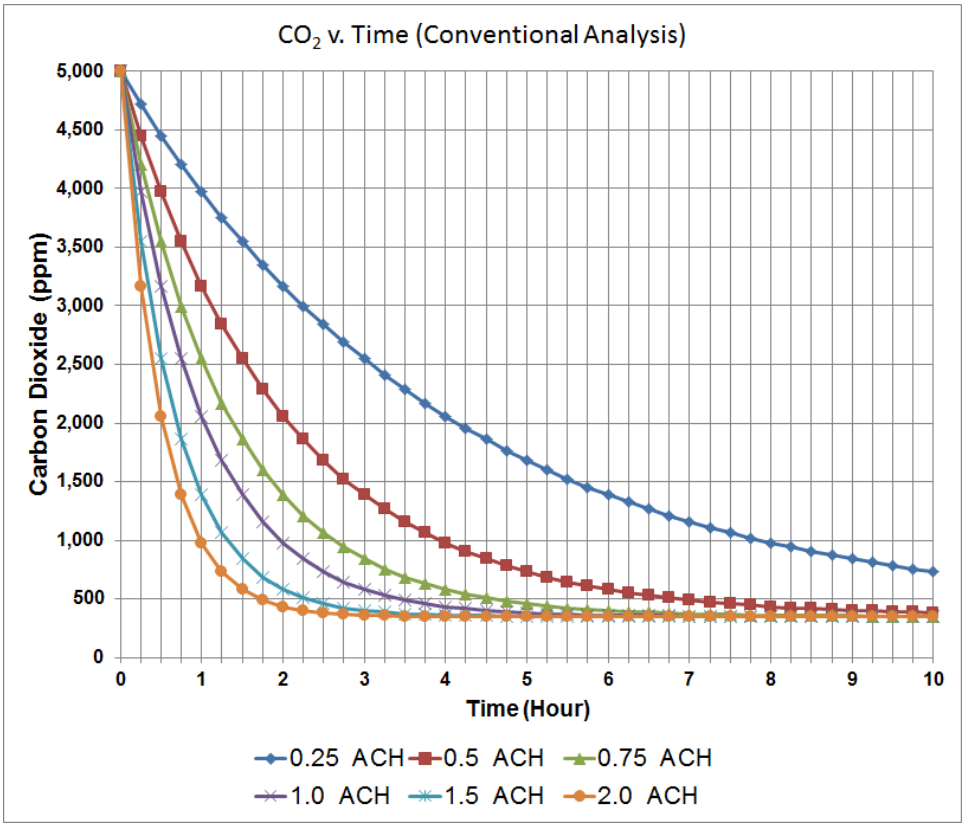


Figure 18

Time	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH
(hr)	0.25	0.25	0.50	0.50	0.75	0.75	1.00	1.00	1.50	1.50	2.00	2.00
	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error
0	0.23	7%	0.46	7%	0.69	8%	0.92	8%	1.37	8%	1.82	9%
	0.23	8%	0.46	8%	0.68	9%	0.90	10%	1.32	12%	1.72	14%
	0.23	8%	0.45	9%	0.67	11%	0.88	12%	1.26	16%	1.58	21%
	0.23	9%	0.45	10%	0.65	13%	0.85	15%	1.17	22%	1.39	30%
1	0.23	9%	0.44	12%	0.64	15%	0.81	19%	1.07	29%	1.17	42%
	0.23	10%	0.43	13%	0.62	17%	0.77	23%	0.94	37%	0.92	54%
	0.22	10%	0.43	15%	0.60	20%	0.72	28%	0.81	46%	0.68	66%
	0.22	11%	0.42	16%	0.57	24%	0.67	33%	0.67	56%	0.48	76%
2	0.22	11%	0.41	18%	0.55	27%	0.61	39%	0.53	65%	0.32	84%
	0.22	12%	0.40	20%	0.52	31%	0.55	45%	0.41	73%	0.21	90%
	0.22	13%	0.39	22%	0.49	35%	0.49	51%	0.31	79%	0.13	93%
	0.22	13%	0.38	24%	0.45	39%	0.43	57%	0.23	85%	0.08	96%
3	0.21	14%	0.37	26%	0.42	44%	0.37	63%	0.16	89%	0.05	97%
	0.21	15%	0.36	29%	0.39	49%	0.31	69%	0.12	92%	0.03	98%
	0.21	16%	0.34	32%	0.35	53%	0.26	74%	0.08	95%	0.02	99%
	0.21	17%	0.33	34%	0.32	58%	0.22	78%	0.06	96%	0.01	99%
4	0.21	17%	0.31	37%	0.28	62%	0.18	82%	0.04	97%	0.01	100%
	0.20	18%	0.30	40%	0.25	67%	0.14	86%	0.03	98%	0.00	100%
	0.20	19%	0.28	43%	0.22	71%	0.12	88%	0.02	99%	0.00	100%
	0.20	20%	0.27	46%	0.19	74%	0.09	91%	0.01	99%	0.00	100%
5	0.20	21%	0.25	49%	0.17	78%	0.07	93%	0.01	99%	0.00	100%
	0.19	22%	0.24	53%	0.14	81%	0.06	94%	0.01	100%	0.00	100%
	0.19	23%	0.22	56%	0.12	84%	0.05	95%	0.00	100%	0.00	100%
	0.19	25%	0.21	59%	0.10	86%	0.04	96%	0.00	100%	0.00	100%
6	0.19	26%	0.19	62%	0.09	88%	0.03	97%	0.00	100%	0.00	100%
	0.18	27%	0.18	65%	0.08	90%	0.02	98%	0.00	100%	0.00	100%
	0.18	28%	0.16	67%	0.06	92%	0.02	98%	0.00	100%	0.00	100%
	0.18	30%	0.15	70%	0.05	93%	0.01	99%	0.00	100%	0.00	100%
7	0.17	31%	0.14	73%	0.04	94%	0.01	99%	0.00	100%	0.00	100%
	0.17	32%	0.12	75%	0.04	95%	0.01	99%	0.00	100%	0.00	100%
	0.17	34%	0.11	77%	0.03	96%	0.01	99%	0.00	100%	0.00	100%
	0.16	35%	0.10	79%	0.03	97%	0.01	99%	0.00	100%	0.00	100%
8	0.16	36%	0.09	81%	0.02	97%	0.00	100%	0.00	100%	0.00	100%
	0.16	38%	0.08	83%	0.02	98%	0.00	100%	0.00	100%	0.00	100%
	0.15	39%	0.08	85%	0.02	98%	0.00	100%	0.00	100%	0.00	100%
	0.15	41%	0.07	86%	0.01	98%	0.00	100%	0.00	100%	0.00	100%
9	0.14	42%	0.06	88%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.14	44%	0.05	89%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.14	46%	0.05	90%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
	0.13	47%	0.04	91%	0.01	99%	0.00	100%	0.00	100%	0.00	100%
10	0.13	49%	0.04	92%	0.00	99%	0.00	100%	0.00	100%	0.00	100%

Analysis with 5-minute intervals:

Time	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH	ACH
(hr)	0.25	0.25	0.50	0.50	0.75	0.75	1.00	1.00	1.50	1.50	2.00	2.00
	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error	$\bar{A}$	Error
0	0.23	7%	0.46	7%	0.70	7%	0.93	7%	1.39	7%	1.85	8%
	0.23	7%	0.46	7%	0.69	8%	0.92	8%	1.38	8%	1.82	9%
	0.23	7%	0.46	8%	0.69	8%	0.92	8%	1.36	9%	1.79	10%
	0.23	7%	0.46	8%	0.69	9%	0.91	9%	1.34	10%	1.76	12%
	0.23	8%	0.46	8%	0.68	9%	0.90	10%	1.32	12%	1.72	14%
	0.23	8%	0.46	9%	0.68	10%	0.89	11%	1.30	13%	1.68	16%
	0.23	8%	0.46	9%	0.67	10%	0.89	11%	1.28	15%	1.64	18%
	0.23	8%	0.45	9%	0.67	11%	0.88	12%	1.26	16%	1.58	21%
	0.23	8%	0.45	10%	0.66	11%	0.87	13%	1.23	18%	1.53	24%
	0.23	8%	0.45	10%	0.66	12%	0.86	14%	1.20	20%	1.46	27%
	0.23	9%	0.45	10%	0.65	13%	0.85	15%	1.17	22%	1.40	30%
	0.23	9%	0.45	11%	0.65	13%	0.84	16%	1.14	24%	1.32	34%
1	0.23	9%	0.44	11%	0.64	14%	0.82	18%	1.10	26%	1.25	38%
	0.23	9%	0.44	12%	0.64	15%	0.81	19%	1.07	29%	1.17	42%
	0.23	9%	0.44	12%	0.63	16%	0.80	20%	1.03	32%	1.08	46%
	0.23	9%	0.44	13%	0.63	17%	0.78	22%	0.99	34%	1.00	50%
	0.23	10%	0.43	13%	0.62	17%	0.77	23%	0.94	37%	0.92	54%
	0.23	10%	0.43	13%	0.61	18%	0.76	24%	0.90	40%	0.84	58%
	0.23	10%	0.43	14%	0.61	19%	0.74	26%	0.85	43%	0.76	62%
	0.22	10%	0.43	15%	0.60	20%	0.72	28%	0.81	46%	0.68	66%
	0.22	10%	0.42	15%	0.59	21%	0.71	29%	0.76	49%	0.61	70%
	0.22	11%	0.42	16%	0.58	22%	0.69	31%	0.71	53%	0.54	73%
	0.22	11%	0.42	16%	0.57	23%	0.67	33%	0.67	56%	0.48	76%
	0.22	11%	0.42	17%	0.57	25%	0.65	35%	0.62	59%	0.42	79%
2	0.22	11%	0.41	17%	0.56	26%	0.63	37%	0.57	62%	0.37	82%
	0.22	11%	0.41	18%	0.55	27%	0.61	39%	0.53	65%	0.32	84%
	0.22	12%	0.41	19%	0.54	28%	0.59	41%	0.49	67%	0.28	86%
	0.22	12%	0.40	19%	0.53	30%	0.57	43%	0.45	70%	0.24	88%
	0.22	12%	0.40	20%	0.52	31%	0.55	45%	0.41	73%	0.21	90%
	0.22	12%	0.40	20%	0.51	32%	0.53	47%	0.37	75%	0.18	91%
	0.22	12%	0.39	21%	0.50	34%	0.51	49%	0.34	77%	0.15	92%
	0.22	13%	0.39	22%	0.49	35%	0.49	51%	0.31	79%	0.13	93%
	0.22	13%	0.39	23%	0.48	36%	0.47	53%	0.28	81%	0.11	94%
	0.22	13%	0.38	23%	0.47	38%	0.45	55%	0.25	83%	0.10	95%
	0.22	13%	0.38	24%	0.45	39%	0.43	57%	0.23	85%	0.08	96%
	0.22	14%	0.38	25%	0.44	41%	0.41	59%	0.20	86%	0.07	97%
3	0.22	14%	0.37	26%	0.43	42%	0.39	61%	0.18	88%	0.06	97%
	0.21	14%	0.37	26%	0.42	44%	0.37	63%	0.16	89%	0.05	97%
	0.21	14%	0.36	27%	0.41	46%	0.35	65%	0.15	90%	0.04	98%
	0.21	15%	0.36	28%	0.40	47%	0.33	67%	0.13	91%	0.04	98%
	0.21	15%	0.36	29%	0.39	49%	0.31	69%	0.12	92%	0.03	98%
	0.21	15%	0.35	30%	0.37	50%	0.29	71%	0.10	93%	0.03	99%
	0.21	15%	0.35	31%	0.36	52%	0.28	72%	0.09	94%	0.02	99%
	0.21	16%	0.34	32%	0.35	53%	0.26	74%	0.08	95%	0.02	99%
	0.21	16%	0.34	32%	0.34	55%	0.25	75%	0.07	95%	0.02	99%
	0.21	16%	0.33	33%	0.33	56%	0.23	77%	0.06	96%	0.01	99%
	0.21	17%	0.33	34%	0.32	58%	0.22	78%	0.06	96%	0.01	99%
	0.21	17%	0.32	35%	0.30	59%	0.20	80%	0.05	97%	0.01	100%

Conclusions:

1) Calculation of "***Air Change Rate***" based on ASTM E741-11, section **8.5.3** will only be accurate if:

   a) Internal generation rate is at or near zero;

   AND

   b) If ambient concentrations of tracer gas are at or near zero;

   Since when using CO_2 "**a**" and/or "**b**" above would be affirmative, then the calculation of "***mean air change rate***" $\bar{A}$ by Equation (ii) would provide an erroneous value:

$$\bar{A} = [\ \ln C(t_2) - \ln C(t_1)\ ]/(t_2 - t_1) \qquad \text{ASTM E741 equation (ii)}$$

The above equation was modified as follows to allow an accurate calculation for the mean air change rate when there are background levels of tracer gas.

$$\bar{A} = [\ \ln[C(t_1) - C_{amb}] - \ln[C(t_2) - C_{amb}]]/(t_2 - t_1) \qquad \text{Equation (ii) modified}$$

2) The Iterative analysis using Eq. #4 – Eq. #10 has been demonstrated to be accurate and directly account for internal generation of tracer gas as well as any ambient concentration of tracer gas.

Error Analyses

CO_2 Detector Error:

Although the typical accuracy of a CO_2 detector is 95%-97% (5%-3% error), the analysis for ACH is based on the difference between CO_2 current and previous trial readings. Such subtractive readings are not subject to a 5%-3% reading error. The theory for ACH error is based on equation #8 and would be as follows:

$$\text{ACH with error} = \frac{[C2*(1+\text{error }\%/100) - \text{Cnew}*(1+\text{error }\%/100)] + C/V}{[C2*(1+\text{error }\%/100) - \text{Camb}]} \bullet \frac{(60\text{ minutes})}{(\text{Interval Minutes})}$$

$$\text{ACH without error} = \frac{[C2*(1+0\ \%) - \text{Cnew}*(1+0\ \%)] + C/V}{[C2*(1+0\ \%) - \text{Camb}]} \bullet \frac{(60\text{ minutes})}{(\text{Interval Minutes})}$$

$$\text{ACH relative error} = \frac{\text{ACH}_{\text{error}} - \text{ACH}_{\text{no error}}}{\text{ACH}_{\text{no error}}} \qquad \text{(Eq. \#13)}$$

Based on equation #10 above, the following relationship between ACH and CO_2 instrument error is as follows:

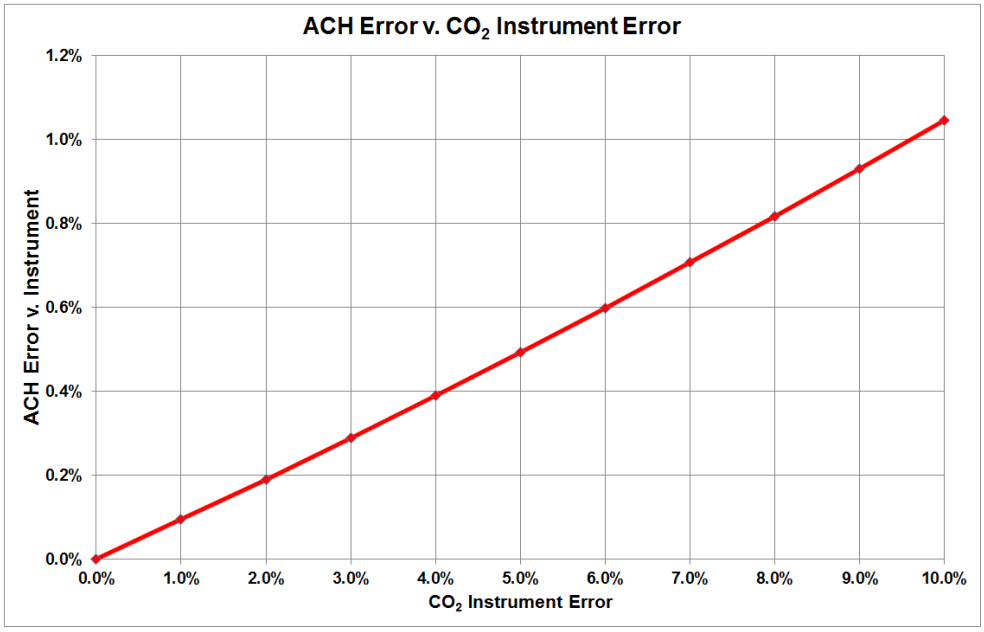


Figure 19

ACH Error v. CO_2 Instrument Error

It can be concluded based on observations from the above that ACH error from instrument error is minimal and would expected to approximately 1% or less.

MET Error Analysis

During testing, the tester or any other person in the room is assigned a MET value of 1.0/person. As an example, if the MET value were actually higher @ 1.2, the MET error would be +20%.

Note: 1 MET = Human activity level at rest; 1 MET = 0.00733 cf of CO_2 per minute.

$$\text{ACH} = \frac{[C(t) - C(t+1)] + C/V}{[C(t) - Camb]} \bullet \frac{(60 \text{ minutes})}{(\text{Interval Minutes})}$$ (Reference Eq. #12 above)

Where: C – cf of CO_2 per time interval; 1 MET = 0.00733 cf per minute per person

Based on the above, and evaluating with a structure with a volume of 12,000 cf (1,500 sf x 8' walls), the relationship for ACH error v. MET error would be as follows:

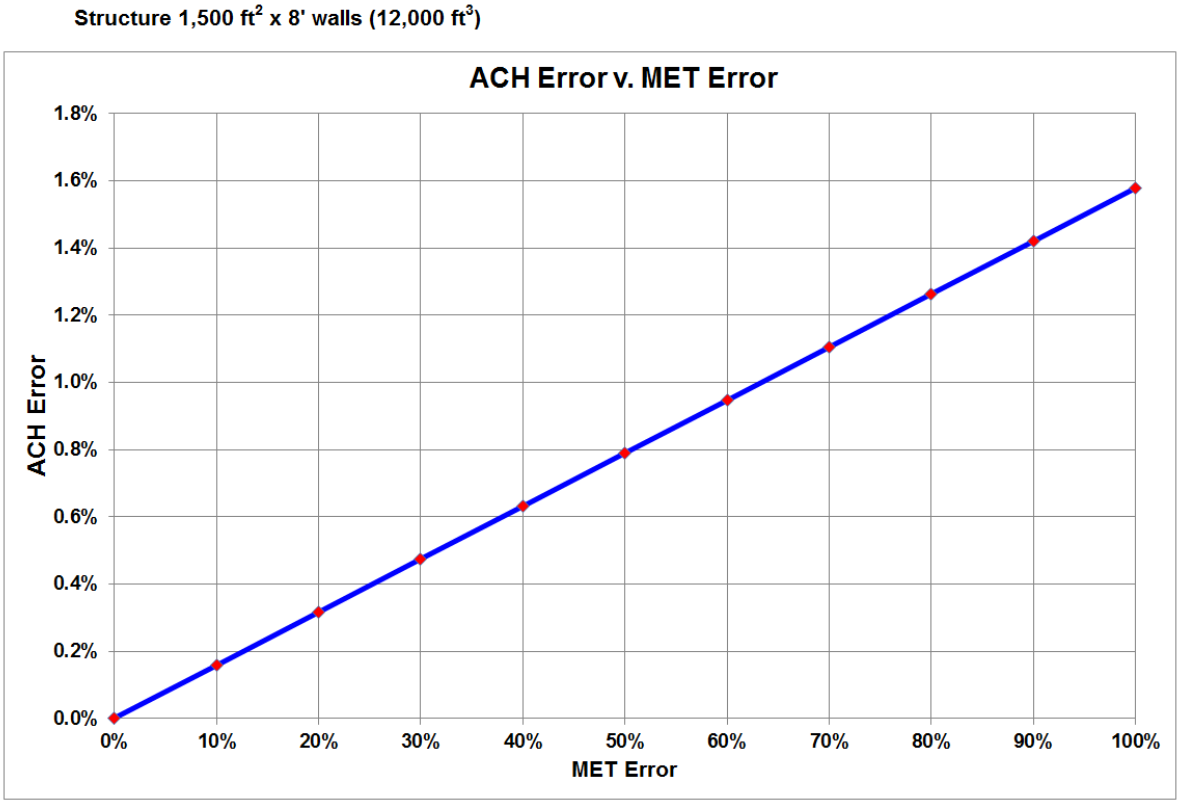


Figure 20
ACH Error v. MET Error

It is reasoned that based on the above analysis, the effects of MET error are minimal due to the high concentrations of CO_2 during testing (~4,000 ppm) that are some ten times normal background levels (~360 ppm).

ACH Error v. MET

A sensitivity analysis was completed for a room volume of 10 m x 10 m x 3m (10,594 cf) using Equation #3 and Equation #5.

CO_2 concentration:

$$\mathbf{C(t) = C_{amb} + G/Q + (C(i) - C_{amb} - G/Q)e^{-It}} \qquad \text{(Eq. \#3)}$$

Air Change Rate:

$$\bar{A} = [\ \ln[C(t_1) - C_{amb}] - \ln[C(t_2) - C_{amb}]]/(t_2 - t_1)$$ modified ASTM E741 equation (ii)

CO_2 Iterative Analysis (5-minute intervals):

$$\mathbf{C_{(t+1)}} = \frac{V \cdot C_t + G + Q \cdot C_{amb} - Q \cdot C_t}{V} = \sum \frac{\mathbf{CO2}}{\mathbf{V}} \qquad \text{(Eq. \#5)}$$

Air Change Rate:

$$\text{ACH} = \frac{[C(t) - C(t+1)] + C/V}{[C(t) - Camb]} \bullet \frac{(60\ \text{minutes})}{(\text{Interval Minutes})} \qquad \text{(Eq. \#10)}$$

The results are as follows:

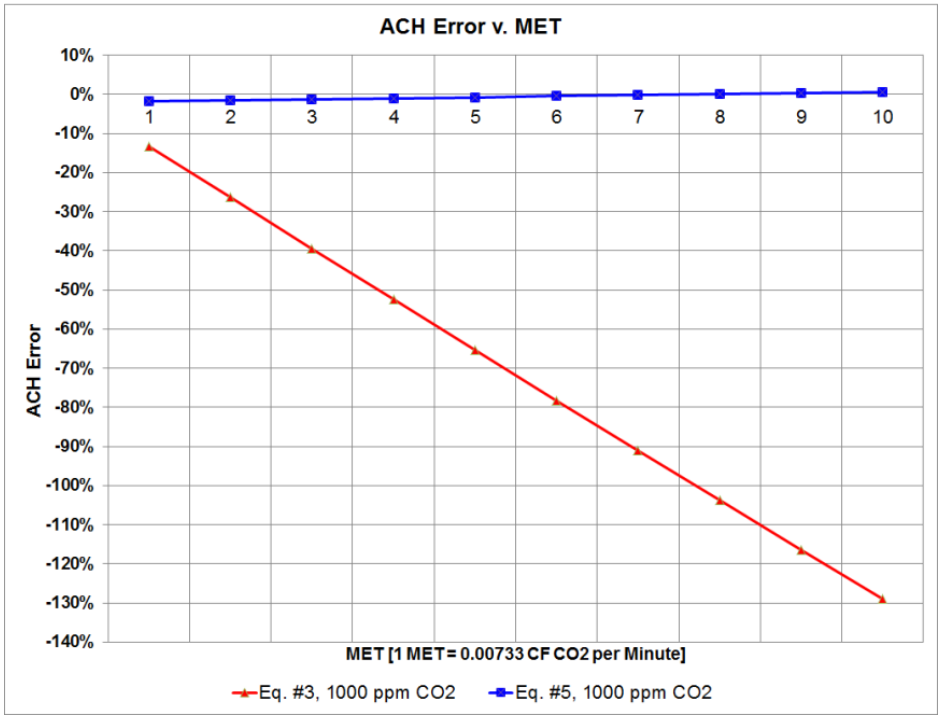


Figure 21

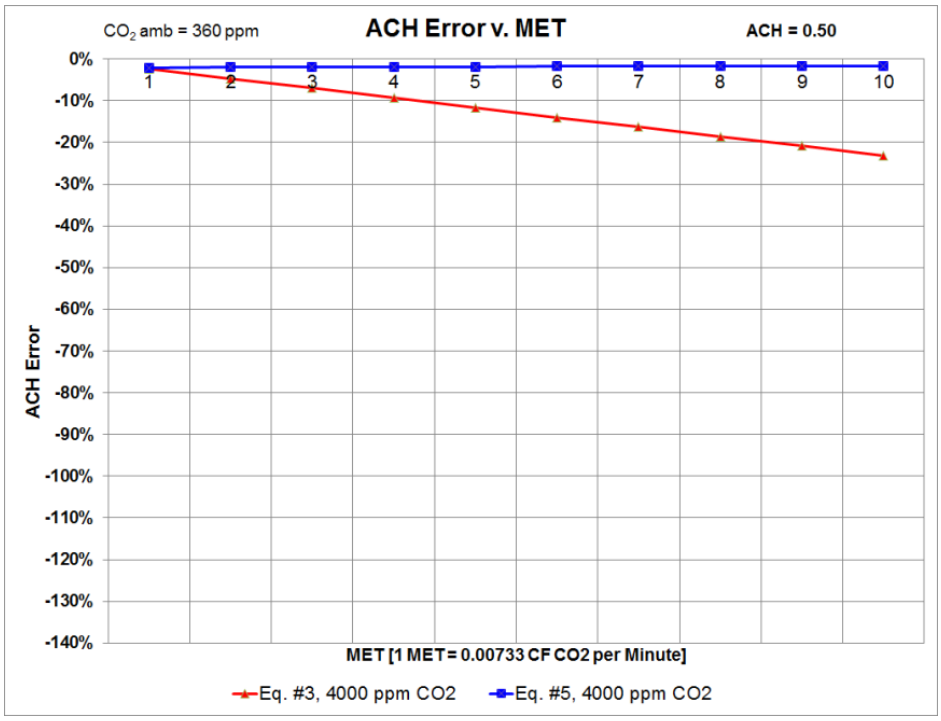


Figure 22

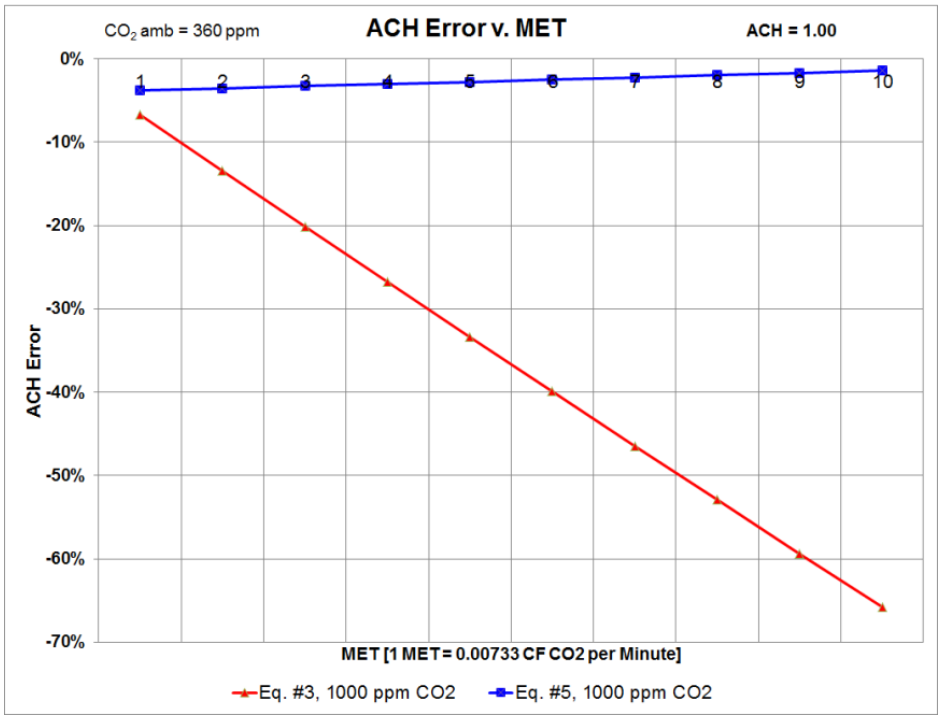


Figure 23

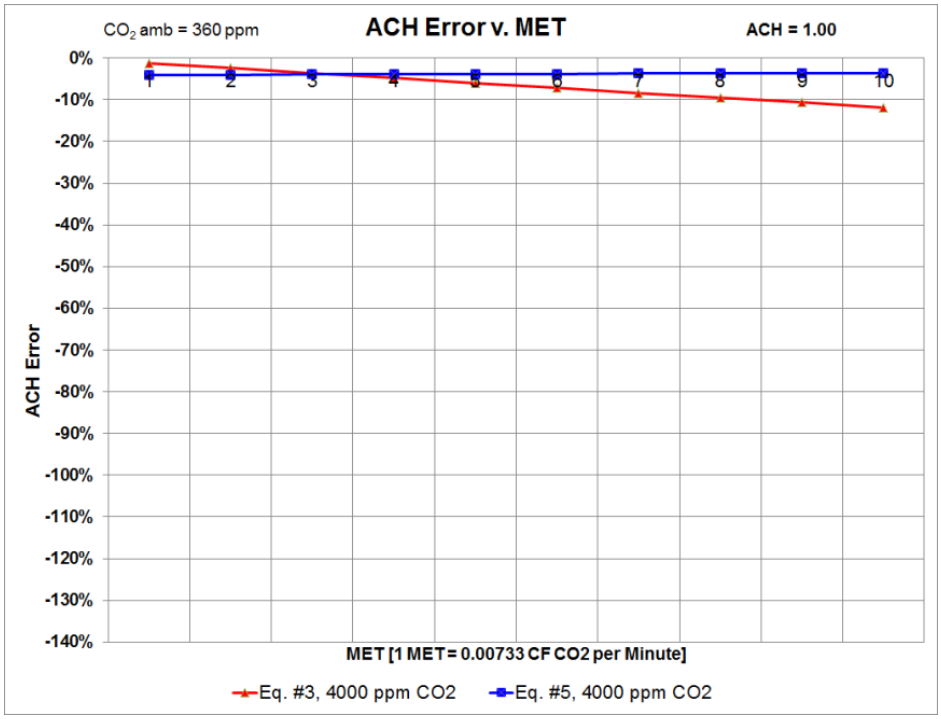


Figure 24

Conclusion

The ASTM E741 Equation (ii) yields large errors exceeding 100% when there is self-generation.

Equation #5 along with Equation #10 even with high levels of self-generation of 1-10 MET can determine air change rates error.

Case Studies

Two air infiltration investigations and a laboratory apparatus were examined to demonstrate the applications for utilizing CO_2 tracer gas testing. The use of infrared camera imagery, and air balance testing are also highlighted to offer additional investigative techniques to the scientist/engineer to enable improved analytic evaluations and more comprehensive opinions.

The subject case studies are as follows:

A) 60,000-square-foot Children's Recreation Facility

The subject children's recreation facility that caught fire was damaged from arson needed an estimated $500,000 in heating, ventilation, and air conditioning (HVAC) code upgrades in order to reopen; the CO_2 tracer gas testing documented naturally occurring building infiltration that offset the code-required mechanical ventilation resulting in reduced capital investment.

B) Urban High Rise Condominium Unit

This urban building experienced major flood damage from a ruptured frozen fire sprinkler pipe that was linked to cold air infiltration into the building from bay windows and exterior adjoining wall surfaces from air stack effects; CO_2 tracer gas testing provided substantiating evidence.

C) Laboratory Test Apparatus

A laboratory apparatus consisted of a PVC gasketed enclosure, CO2 monitor, miniature circulation blower, airflow meter, and miniature intake blower allows a controlled determination of air infiltration rate.

Use of Tracer Gases to Determine Air Infiltration and Other Applications

CO_2 tracer gas techniques, such as the ones used in these investigations, also have important applications in analyzing heating and cooling loads, modeling carbon monoxide (CO) or other contaminant concentrations, and evaluating air quality in industrial, commercial, and residential structures.

Note that the purpose of tracer gas testing has been well established by the following:

a) ASTM E741, Standard Test Method for Determining Air Change in a Single Zone by Means of a Tracer Gas Dilution.[3]

b) ASTM D6245, Standard Guide for Using Indoor Carbon Dioxide Concentrations to Evaluate Indoor Air Quality and Ventilation.[4]

The above forensic engineering investigations used CO_2 to determine air infiltration. These evaluations showed that the measurement of tracer gases could detect concentrations of air contaminants and, cold air excess air infiltration placing water pipes at risk, as well as health/safety code violations. Using CO_2 as a tracer gas can be an effective tool for forensic engineers, HVAC engineers, or other facilities engineers to evaluate air infiltration. An experienced engineer using CO_2 tracer gas testing could detect possible problems or point to potentially dangerous situations in ways that are non-destructive, safe, and economical making these methods acceptable to building owners as well as mitigating financial liability from subsequent building damage events.

This paper establishes an improved and more straightforward mathematical methodology and calls for establishing initial CO_2 levels of 4,000 – 5,000 ppm to enable more accurate test results.

There are several forensic investigation applications where there is a need to determine the air infiltration in a structure or enclosure. Some examples would include, but may not be limited to**,** the following:

a) Building freeze analysis,
b) Heating/cooling load analysis,
c) Carbon monoxide,
d) Explosions caused by a volatile gas,
e) Children/pets trapped in a hot car,
f) Persons trapped in elevators, and
g) Air quality evaluation.

The choice of CO_2 as a tracer gas was made due to the more practical aspects of utilizing CO_2:

- CO_2 gas is non-toxic and safe to use with minimal risk with proper protocol,
- CO_2 is more acceptable to building owners,
- CO_2 detector instrumentation is readily available and economical ($200 - $1,000),
- CO_2 sold in 5 lb, 20 lb, and 50 lb tanks @ $1/lb ($20/ typical home), and
- CO_2 has a specific weight only 51% greater than air.

Air Infiltration

Air infiltration is caused by mechanical ventilation, and the stack effect of warm air rising through a building as well as the wind pressure gradient (see Figures 5 and Figure 6 below.).

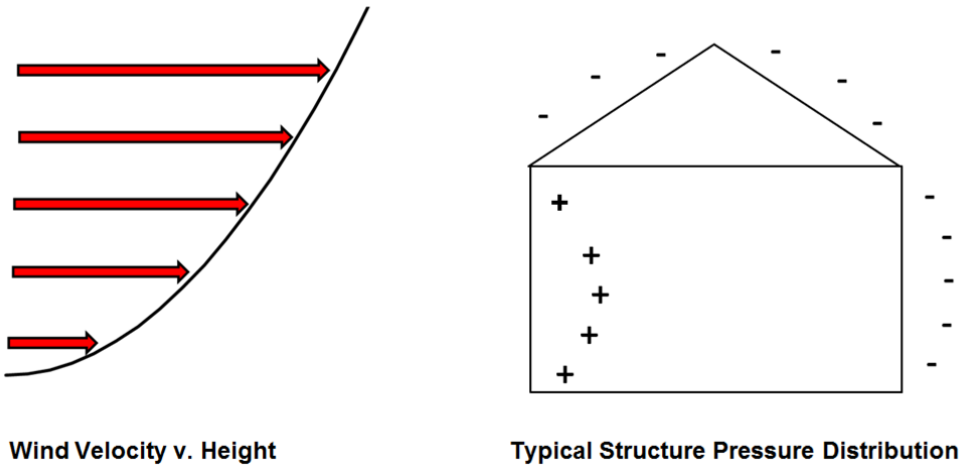


Figure 25
Infiltration from Wind Pressure

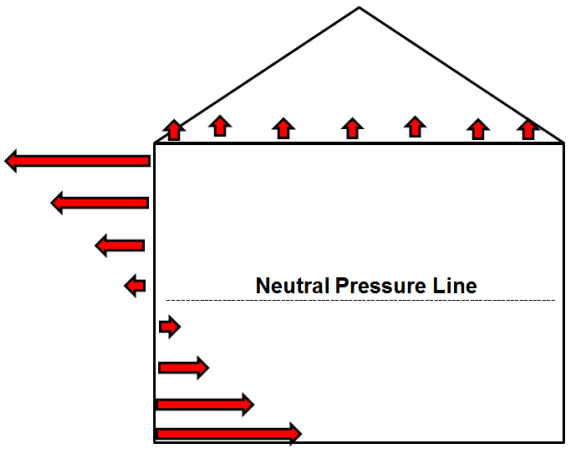


Figure 26
Infiltration from Buoyant Air Pressure

In multi-story structures, the stacking effect is cumulative based on the height of the building; typically, the taller the structure, the greater the stack effect; see Figure 7 and Figure 8 below.[8]

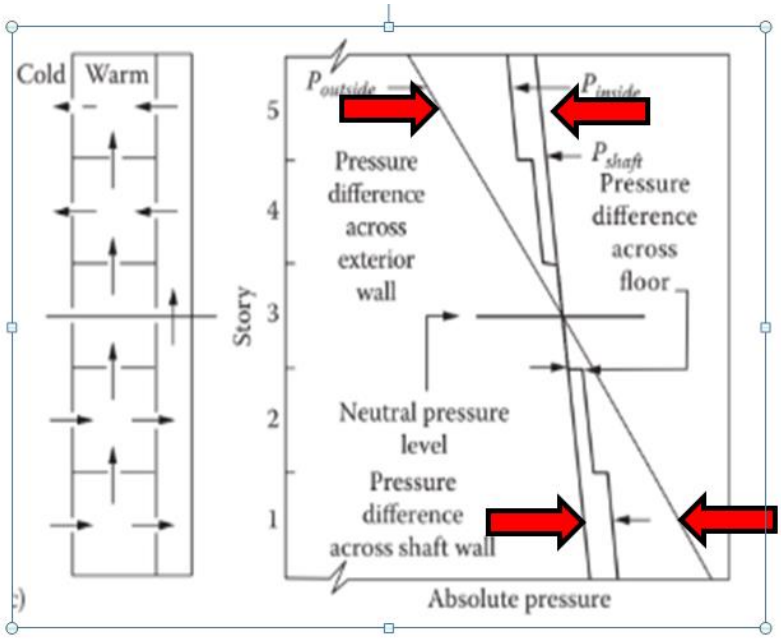


Figure 27
Multi-story Stack Effect

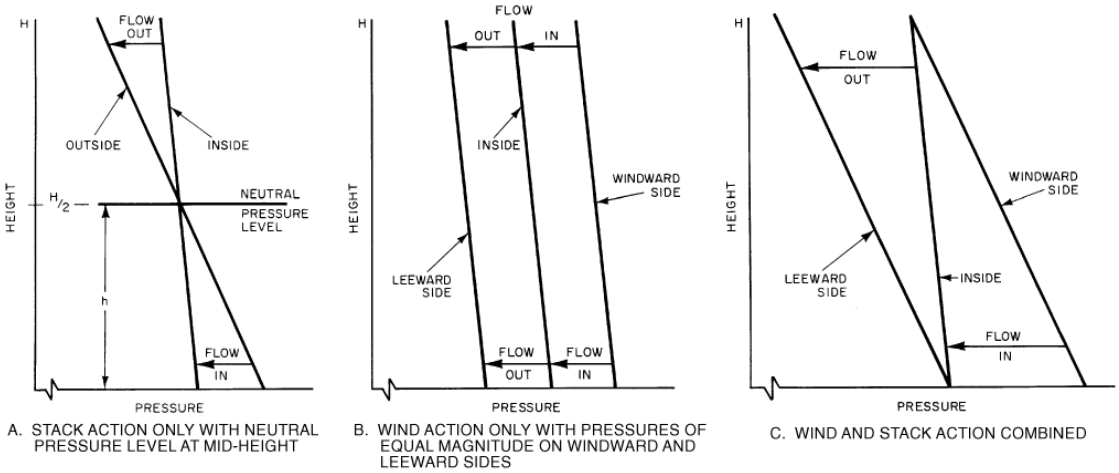


Figure 28
Combined Effect, Stack + Wind Infiltration

Copyright notice: ©ASHRAE www.ashrae.org (ASHRAE Fundamentals,[8] 2005).

Based on ASHRAE Fundaments (2005)[8], Residential Ventilation, 27.15, the statistical values for infiltration are represented below:

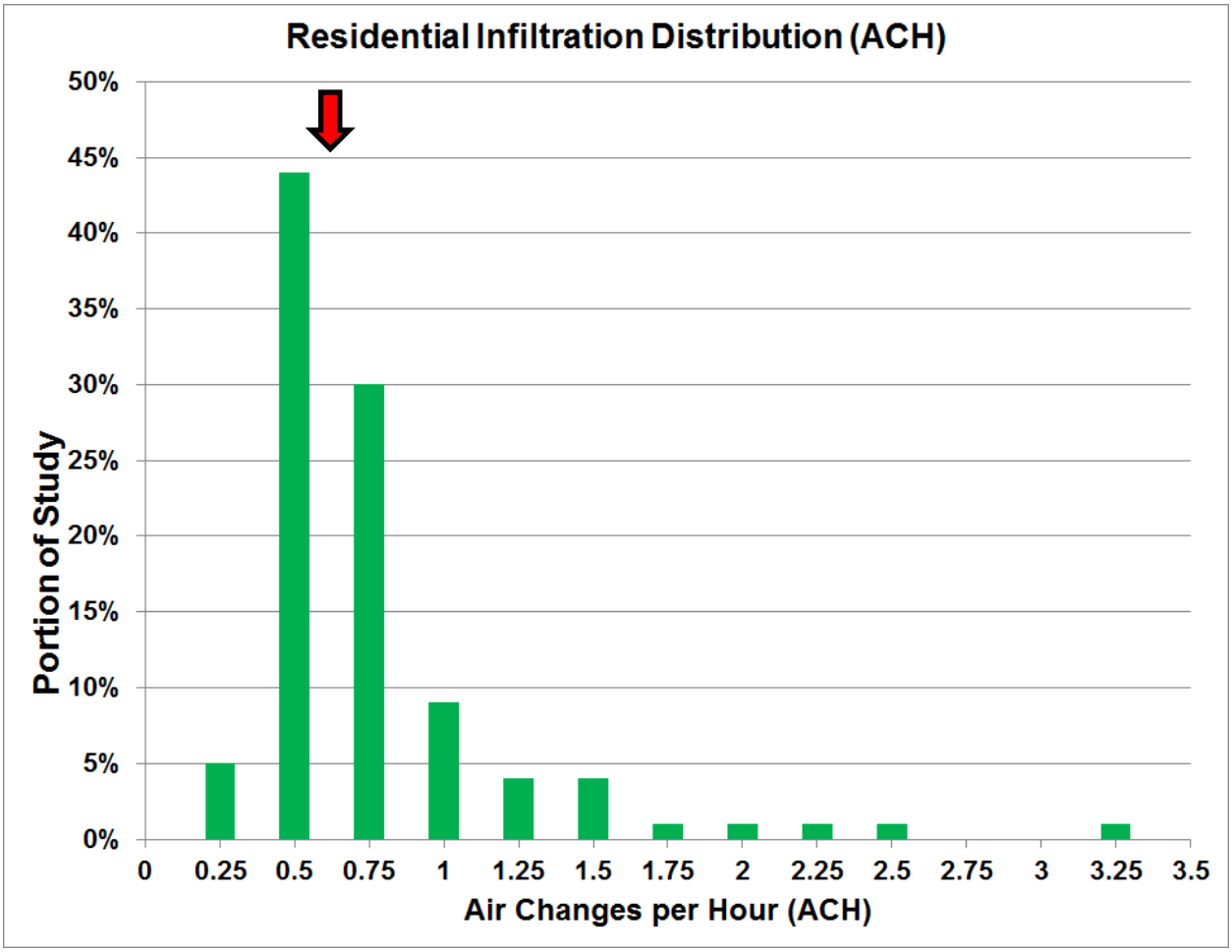


Figure 29
Histogram of Infiltration Values – New Construction

Note: Red Arrow: Statistical mode.

Copyright notice: ©ASHRAE www.ashrae.org (ASHRAE Fundamentals[8], 2005).

Blower Door Testing

Blower door testing[9] determines the overall infiltration tightness of wall/ceiling structures with a 50 Pa differential pressure (0.20" H_2O, equivalent to 20 mph wind) on typically freestanding residential structures. Such testing is usually performed with the HVAC, and appliance ductwork blocked off and with testing conducted in calm wind conditions. A blower door test utilizes a blower fan that has been positioned within a sealed-off exterior door opening. The fan exhausts air from the home/structure creating a negative pressure. The outside air then passes through any unsecured cracks and openings.

Figure 30
Typical Blower Door Test Apparatus

The ACH_{50} value has a volumetric air flow @ 50 Pa in the numerator and a building volume in the denominator. The ratio of ACH_{50} cubic feet/hr per cubic foot (cf) provides an index of the air leakage per unit volume for a typical home; the test is more

commonly conducted on freestanding structures. The ACH_{50} test is based on 50 Pa (0.20" H_2O) omnidirectional negative differential air pressure:

$$ACH_{50} = \frac{CFM_{50Pa} \times 60 \text{ min/hr}}{\text{[Building Volume]}}$$

Based on standards established by ASHRAE[8] and The Energy Conservatory (TEC),[9] (manufacturers of the ***Minneapolis Blower Door***™), the following comparison is presented below:

Blower Door Testing		Stack Effect
Air Changes per hour @ 50 Pa		Air Changes per hour
ACH_{50}	**Ranking**	**ACH**
0.0 - 1.5	Very Tight	0-20%
1.5 - 3.0	Tight	20%-40%
3.0 - 5.0	Moderately Tight	40%-60%
5.0 - 7.0	Loose	60%-80%
7.0 - 10.0	Very Loose	80%-125%
10 +	Extremely Loose	125% +

Figure 31
Blower Door and Stack Effect Comparison

Note 1: The blower door test best practices may include an evaluation of air leakage rates @ 75 Pa (ACH_{75}) for high-rise buildings due to greater stack effects resulting from greater building height.

Note 2: The above values may not be based on concrete slab floors and ceilings or perimeter interior walls.

Tracer Gas Comparison

The table below summarizes known tracer gases, monitor costs, and other physical properties and considerations for tracer gas testing:

Item	Description	Gas Readily Available	Monitor Cost	Dectector Portability	Radioactive, Toxic, Flammable, or Irritant	Molecular Weight	Specific Weight
1	Acetone (C_3H_6O)	Yes	>$50,000	Yes	Yes	58.07	1.9938
2	Radioactive Argon (^{41}AR)	No	>$10,000	No	Yes	39.95	1.3716
3	**Carbon dioxide (CO_2)**	**Yes**	**$ 500**	**Yes**	**No**	43.99	**1.5104**
4	Helium (He)	Yes	>$10,000	Yes	No	4.00	0.1374
5	Radioactive Krypton (^{85}KR)	No	>$10.000	No	Yes	83.80	2.8772
6	Methane (CH_4)	Yes	>$25,000	Yes	Yes	16.04	0.5508
7	Nitrous Oxide (N_2O)	Yes	>$25,000	Yes	Yes	44.00	1.5108
8	Perfluorodimethylcyclohexane (PDCH)(C_8F_{16})	No	>$50,000	Yes	No	400.06	13.7354
9	Perfluoromethylcyclohexane (PMCH)(C_7F_{14})	No	>$50,000	Yes	No	350.05	12.0184
10	Sulfur hexafluoride (SF_6)	Yes	>$32,000	Yes	Minimal	146.05	5.0145
11	Trifluoromethyl sulfur pentafluoride (SF_5CF_3)	No	>$50,000	Yes	Yes	196.06	6.7314

Figure 32
Tracer Gas Comparison

Conclusion:

Based on the above data**,** CO_2 is the best choice for tracer gas testing due to the lowest cost for gas detectors and gases as well as the availability of both gases and detectors.

CO_2 controllers have the following advantages:

a) Lowest price,
b) Fully portable,
c) Accurate, and
d) Easy to operate; data is observed by simply turning instruments on; there is no procedural protocol.

CO_2 gas has the following advantages:

a) Non-toxic when applied in concentrations of less than 5,000 ppm,
b) Safety protocols include the use of gloves and eye protection,
c) CO_2 gas is competitively priced and readily available, and
d) The CO_2 specific weight of 1.5104 is only 10% more than Argon and slightly lower than Nitrous Oxide (1.5108); buoyancy problems are decreased.

Testing Protocol

a) Use fans/blowers for mixing/recirculation of air,

b) Establish initial levels of 4,000 – 5,000 ppm (to minimize fluctuations in background CO_2),

c) Release CO_2 into fans for distribution,

d) Confirm concentration by CO_2 monitor,

e) Confirm 0.45% CO_2 dose by space volume, and tare weight of CO_2 using calibrated weight scales, and

f) Take readings every 5 minutes manually or with data logging instrumentation.

Note: Account for CO_2 of test engineer, use remote sensor, or vacate space during testing.

Safety Protocol

a) Use thermally insulated gloves and eye protection when handling tanks and CO_2,
b) Release CO_2 into blower for rapid mixing into air,
c) Establish initial CO_2 levels of 4,000 – 4,500 ppm,

Note: Permissible 8 hr exposure limits are 5,000 ppm
Maximum safety limits are 10,000 – 15,000 ppm

d) Confirm concentration by CO_2 monitor,
e) Confirm 0.45% CO_2 dose by space volume, and tare weight of CO_2 using calibrated scales,
f) Best testing in vacant building spaces, and
g) Prohibit testing involving any at-risk populations.
(i.e.: children, elderly, pregnant women, etc.).

h) Follow established Material Safety Data Sheet guidelines, please see **Appendix A**, Material Safety Data Sheet for CO_2.

Air Change per Hour (ACH) Graphs/Charts

The charts below are based on a structure that is: 2.5 m high, 100 m^2 in floor area, and 250 m^3 in volume (Equivalent to 10,764 sf, 8.2ft high, and 8,825 cf).

Please see Figures #33- #37 below:

Figure 33

CO_2 v. Time v. ACH (0.10 to 3.0) **[MET = 1.0]**

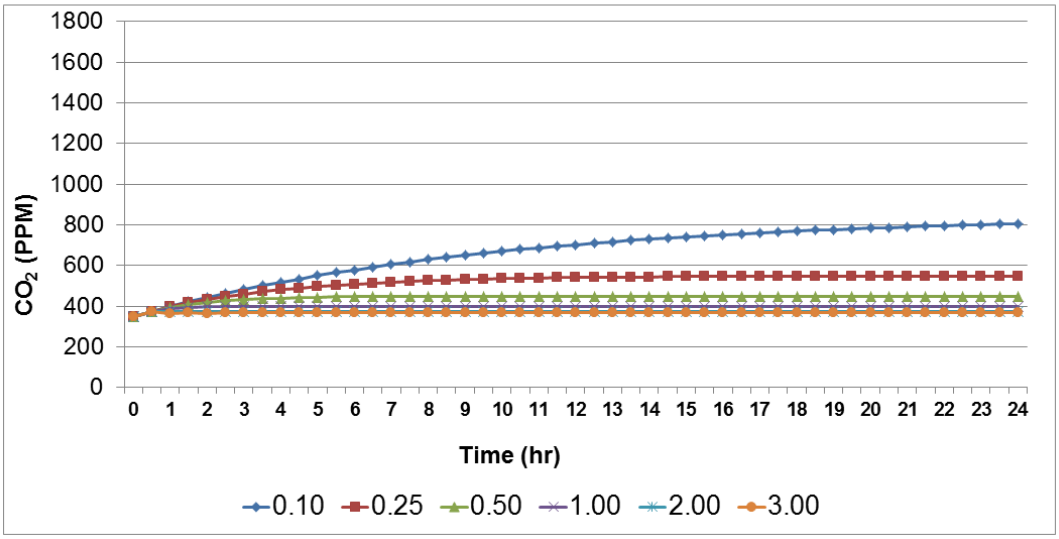


Figure 34

CO_2 v. Time v. ACH (0.10 to 3.0) **[MET = 2.0]**

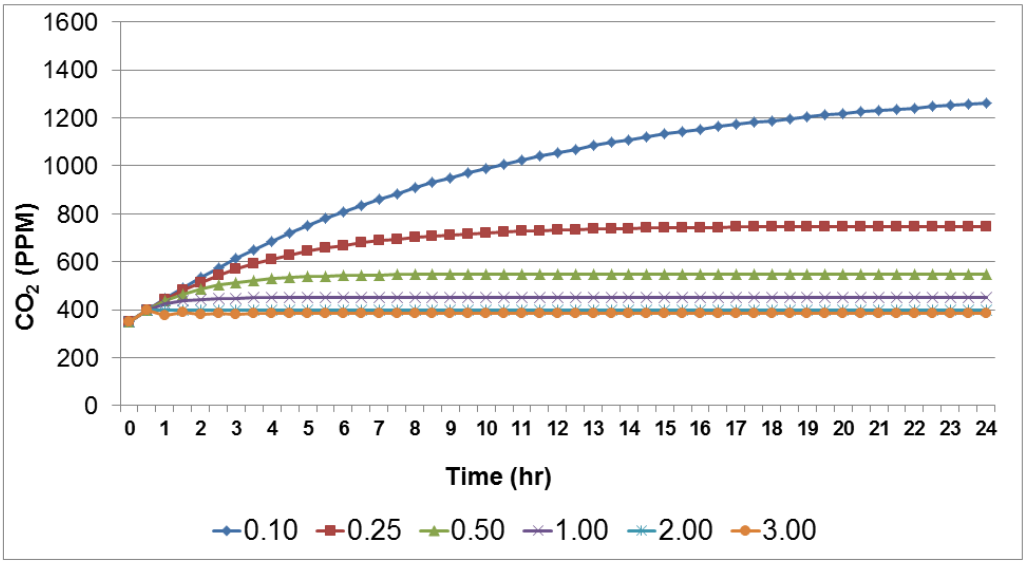


Figure 35

CO_2 v. Time v. ACH (0.10 to 3.0) **[MET = 3.0]**

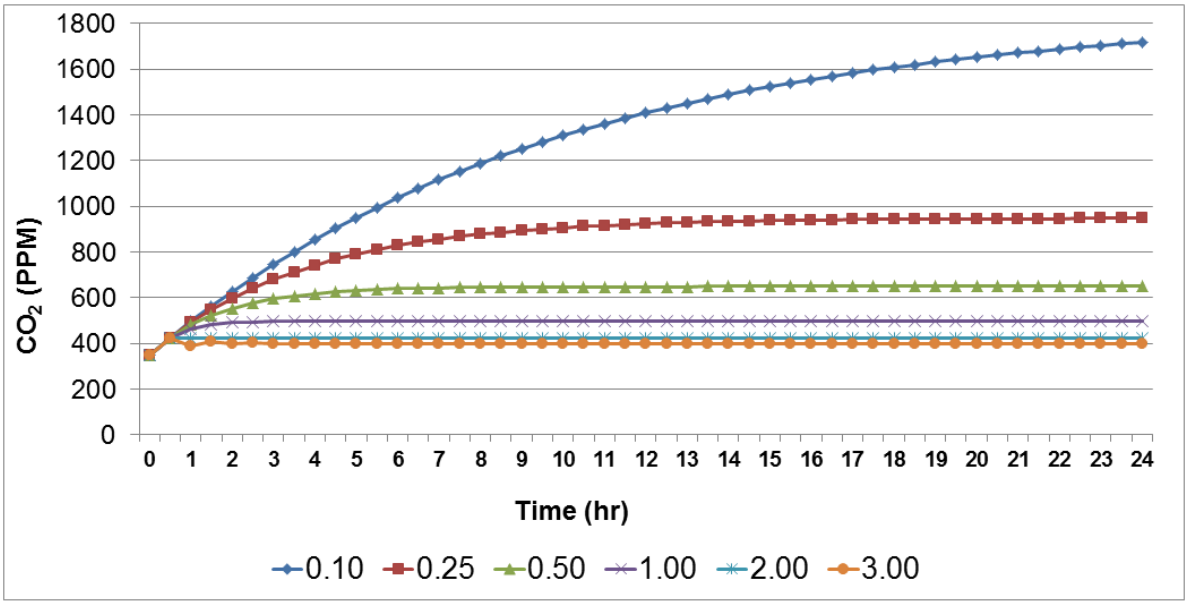


Figure 36

CO_2 Concentration v. ACH

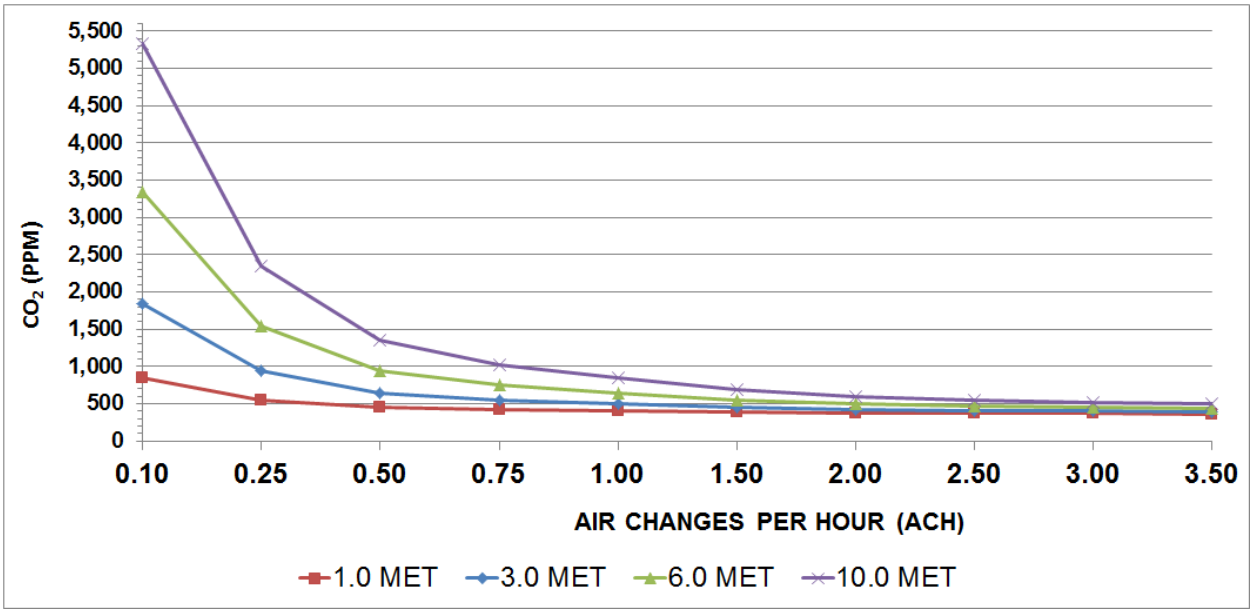
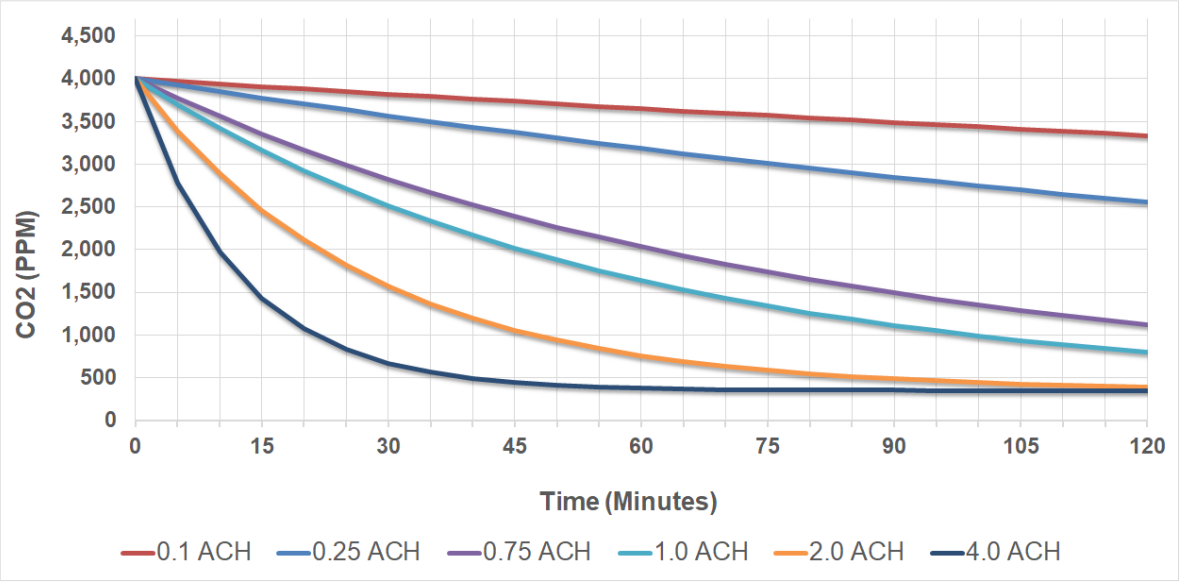


Figure 37



Note:

The above analysis charts provide a visual representation of the mathematical models of CO_2 concentration v. ACH (air changes per hour) and provide a useful tool for the forensic investigative engineer.

BUILDING ENVELOPE AND PIPE-FREEZING CODE STANDARDS

The following code standards are applicable for building envelope and pipe-freezing protection best practices:

2015 INTERNATIONAL PLUMBING CODE[11]

305.4 Freezing. Water, soil, and waste pipes shall not be installed outside of a building, in attics or crawl spaces, concealed in outside walls, or in any place subjected to freezing temperatures, unless adequate provision is made to protect such pipes from freezing by insulation, heat, or both.

2015 INTERNATIONAL ENERGY CONSERVATION CODE[12]

R402.4 Air leakage (Mandatory)

The building thermal envelope shall be constructed to limit air leakage in accordance with the requirements of Sections R402.1 through R402.4

Table 402.4.1.1 Air Barrier and Insulation Installation General Requirements.

A continuous air barrier shall be installed in the building envelope. The exterior thermal envelope contains a continuous air barrier. Breaks or joints in the air barrier shall be scaled.

Air-permeable insulation shall not be used as a sealing material.

802.3.3 Sealing of the building envelope. Openings and penetrations in the building envelope shall be sealed with caulking materials or closed with gasketing systems compatible with the construction materials and location. Joints and seams shall be sealed in the same manner or taped or covered with moisture vapor-permeable wrapping material. Sealing materials spanning joints between construction materials shall allow for expansion and contraction of the construction materials.

NFPA 25, Standard for the Inspection, Testing, and Maintenance of Water-Based Fire Protection Systems (2017)[13]

4.4.1.2

In areas that have the potential for freezing temperatures below the level that can be adequately protected by an allowable antifreeze solution, supplemental heat can be provided when temperatures fall below the level of the antifreeze solution. Other means of freeze protection for water-filled piping, including heated vale enclosures, heat tracing, insulation, or other methods, are allowed by the applicable installation standards. Installation standards require heat tracing protecting fire protection piping against freezing to be supervised.

4.1.2.1

All areas of the building containing water-filled piping that do not have another means of freeze protection shall be maintained at a minimum temperature of 40°F (4.0°C).

4.1.2.2

Aboveground water-filled pipes that pass through open areas, cold rooms, passageways, or other areas exposed to temperatures below 40°F (4.0°C), protected against freezing by insulating coverings, frost-proof casings, listed heat tracing systems, or other reliable means, shall be maintained at temperatures between 40°F (4.0°C) and 120°F (48.9°C).

4.1.2.3

Where other approved means of freeze protection for water-filled piping, as described in 4.1.2.2, are utilized, they shall be inspected, tested, and maintained in accordance with this standard.

Case Study A

60,000-Square-Foot Children's Recreational Facility

Background

The following case involves a 60,000-sf children's recreational facility that was damaged from a fire caused by arson (see Photo C1 below). It was determined that the building needed to have code upgrades to meet requirements established by the local building authority and international code standards. One of the primary issues was that there needed to be a make-up air system and exhaust air system to satisfy ventilation requirements to meet the needs of the expected occupancy of over 500 persons. The costs for the make-up air system were estimated at over $500,000. It was proposed to utilize the normally occurring building infiltration to offset the need for mechanical ventilation equipment to satisfy building code requirements. CO_2 tracer gas testing was conducted to determine the naturally occurring building infiltration. The plan also called for the installation of a make-up air unit and a CO_2 monitor to regulate the exhaust fans if CO_2 levels reached levels of over 900 ppm.

A detailed review of the HVAC requirements at the recreational facility located in the Northeast was completed.

The goal of the engineering assignment was to provide a set of recommendations for the HVAC system to meet both the health needs of the public and be consistent with standards of good engineering practice.

Figure A1

Subject Children's Recreational Facility

The subject facility has the following use areas:

-Party Area (64' x 56' ~ 3,584 sf)

-Laser Tag (64' x 46' ~ 2,944 sf)

-Main Area (150' x 102'~15,300 sf), [including kitchen, bathrooms, and administration offices/areas] (see photo C2 below).

Figure A2
Main Party Play Area (Note: Ceiling fans above provide air circulation)

The ceiling height ranges from 20'-6" to 29'-8" with an average of approximately 25'.

The facility is presently heated with natural gas including a 300,000 Btu/hr hydronic boiler and a radiant floor distribution system with two zones.

There are three 3-ton heating/cooling rooftop units servicing the administration offices, kitchen, and party room.

There are four dedicated air-conditioning units:

Main Area: Two – 12 ton
Party Room: One - 7.5 ton
Laser Tag: One – 7.5 ton

There are also four roof exhaust fans:

Exhaust #1: 1,600 cfm (Adjacent to the manager's office)
Exhaust #2: 1,000 cfm (Between #1 and #3)
Exhaust #3: 1,600 cfm (Adjacent to the bathrooms)
Exhaust #4: 1,600 cfm (Party area)

Historic seasonal peak occupancies of the spaces were determined as follows:

Winter:

Main Area:	400
Party Room:	100
Laser Tag:	20
	520 persons

Summer:

Main Area:	150
Party Room:	100
Laser Tag:	20
	270 persons

A CO_2 tracer gas test was completed to determine the infiltration of the building. To administer the test, approximately 200 lb of CO_2 were released into the main area (Laser Tag and Party areas were closed off with a tarpaulin). Approximately 20 floor-mounted blower fans were positioned to rigorously mix the air in the main area (see photos on pages 6 and 7). Testing was completed in mid-December in the mid-afternoon; outside wind conditions were calm; outdoor temperatures ranged from 35ºF to 40ºF. The documentation of the building infiltration was then part of the air-balance consideration for the space.

Figure A3

CO_2 Tracer Gas Testing, [Note CO_2 cylinders and circulation fans (indicated).]

Infiltration test results were as follows:

Time [min]	CO2 (PPM)	ACH (Air Changes/hr)
0	3,600	
10	3,409	0.38
20	3,262	0.30
30	3,190	0.15
40	3,104	0.19
50	3,019	0.19
60	2,942	0.18
70	2,862	0.19
	AVE:	0.26

				Infiltration	Infiltration		
		AVE	3000	A.C.H. @	A.C.H. @		
	Floor	Ceil	Volume	0.26	0.26		Volume
	[sf]	[ft]	[cf]	[cf/hr]	[cfm]		
Main space	15,300	25.0	381,930	99,302	1,655		70.1%
Party room	3,584	24.9	89,216	23,196	387		16.4%
Laser tag	2,944	25.0	73,728	19,169	319		13.5%
	21,828		544,874	141,667	2,361		100.0%

Summer Cooling Mode:

						Summer	Required		
					People @	Max	Air		A.C.
	Infiltration		Exhaust	Total	15 cfm	People			O.A.
	[cfm]		[cfm]	[cfm]			[cfm]		[cfm]
Main space	1,655		1,437	3,092	206	150	0		2,160
Party room	387		336	722	48	100	778		675
Laser tag	319		277	597	40	20	0		675
	2,361		2,050	4,411	294	270	778		3,510

Winter Heating Mode:

						Winter	Additional
					People @	Max	Required
	Infiltration		Exhaust	Total	15 cfm	People	Air
	[cfm]		[cfm]	[cfm]			[cfm]
Main space	1,655		1,437	3,092	206	400	2,908
Party room	387		336	722	48	100	778
Laser tag	319		277	597	40	20	0
	2,361		2,050	4,411	294	520	3,686

Use of CO_2 Controls (See figure below)

CO_2 controls were utilized in the Party room and Main space to control the existing exhaust fans #1, #3, & #4.

Figure A4
CO_2 Controller

Use of Ceiling Fans:

56” ceiling fans were installed in the following areas to better allow the distribution of air:

Laser Tag: 2 fans Party Room: 4 fans Open Play Area: 20

Make-up Air Units

Two 2,500 cfm make-up air units (totaling a minimum of 5,000 cfm) were installed to provide needed make-up air and additional heat for the space. The make-up air units featured a modulating gas valve and the units were linked to CO_2 controls. The units were mounted on the mezzanine over the bathrooms and distributed air into the Main space and Party room.

It was also proposed that the damper position of the four roof-top units be interlocked with the CO_2 controller to enable a proper air balance when the building needed additional ventilation.

Figure A5, Subject make-up air units.

Figure A6, with access panels removed.

Case A Conclusions:

a) New make-up air equipment was installed that included: ceiling fans, CO_2 controls, and make-up air.

b) Natural air infiltration was utilized to offset the need for mechanical system make-up air.

c) CO_2 monitors were used to control roof exhaust fans to maintain proper air quality during peak occupancies.

d) The engineered approach saved the insurance carrier more than **$250,000** on HVAC mechanical systems.

Case Study B

Urban High Rise Condominium Unit

Background Information:

A condominium unit in a high-rise building in a northeastern city experienced major flood damage from a ruptured frozen fire sprinkler pipe. An investigation was conducted to determine if the causation was from cold air infiltration; (see Figures B1- B22 below).

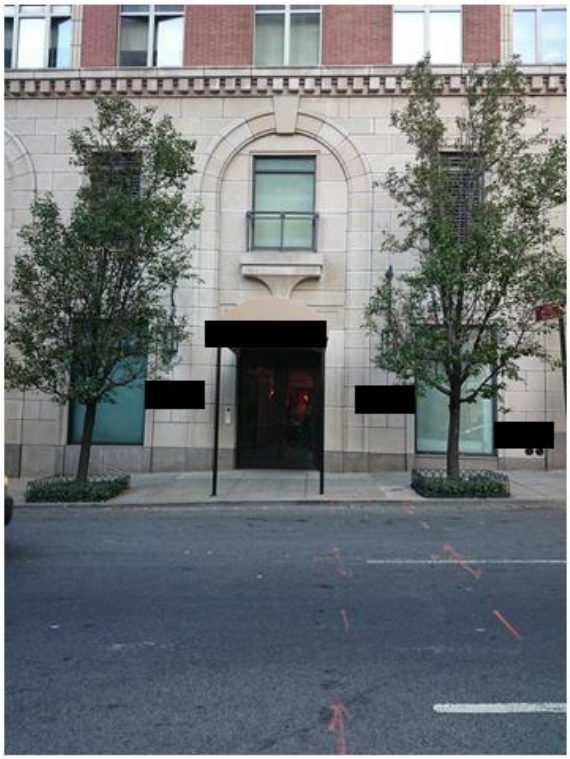


Figure B1
View from Street

Figure B2
View of Bay Window

Figure B3

View of Upper Building Floors

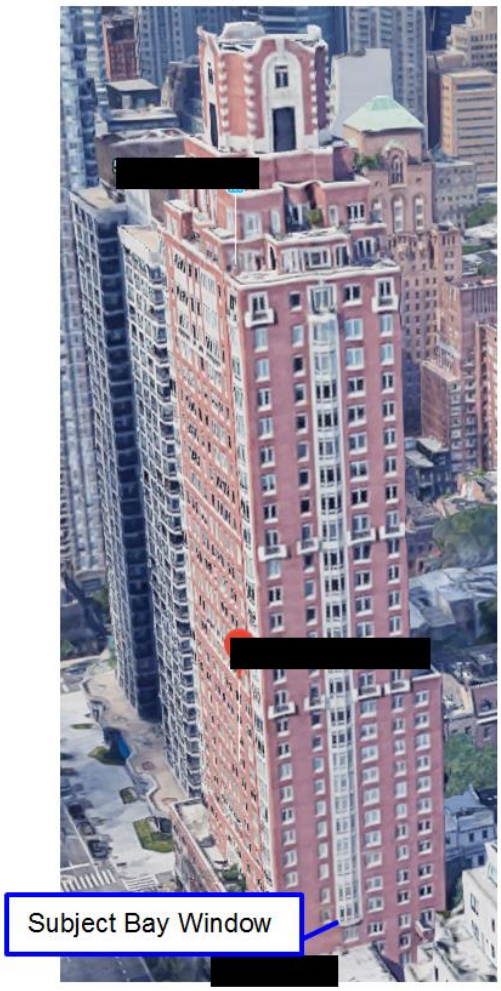
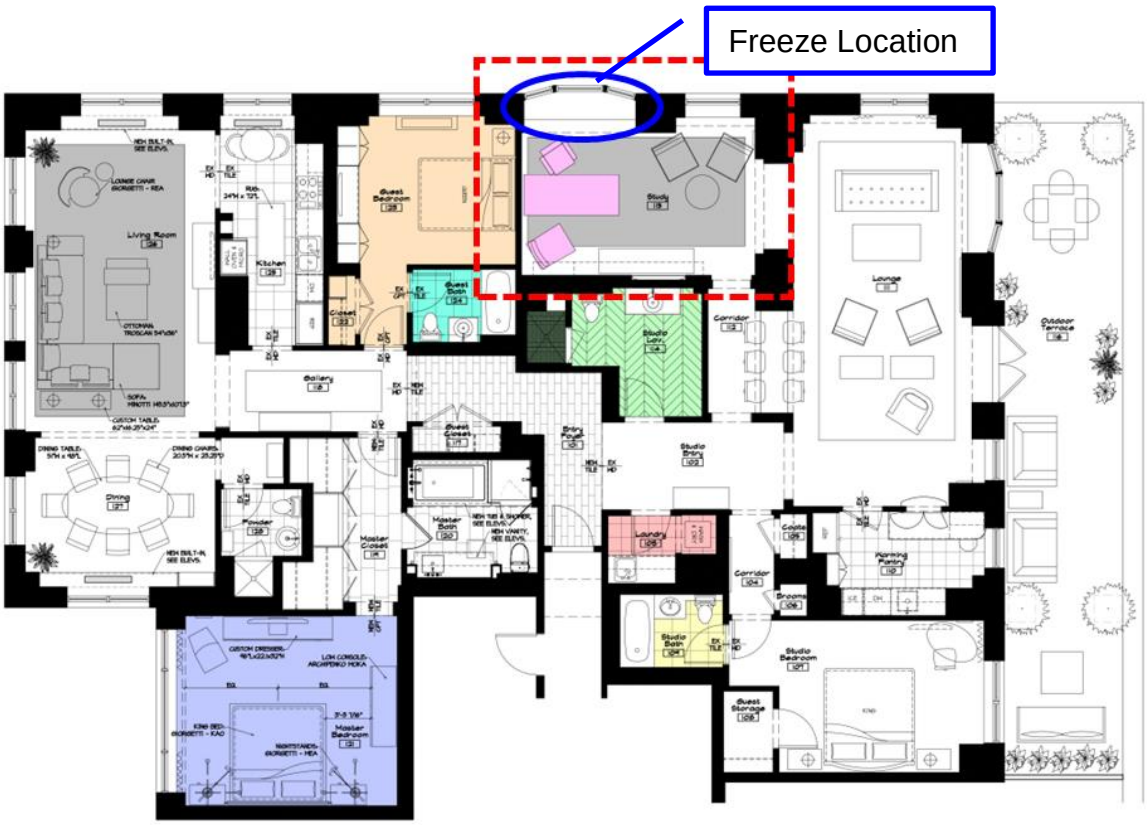


Figure B4

Subject Bay Window

Subject Floor Plan



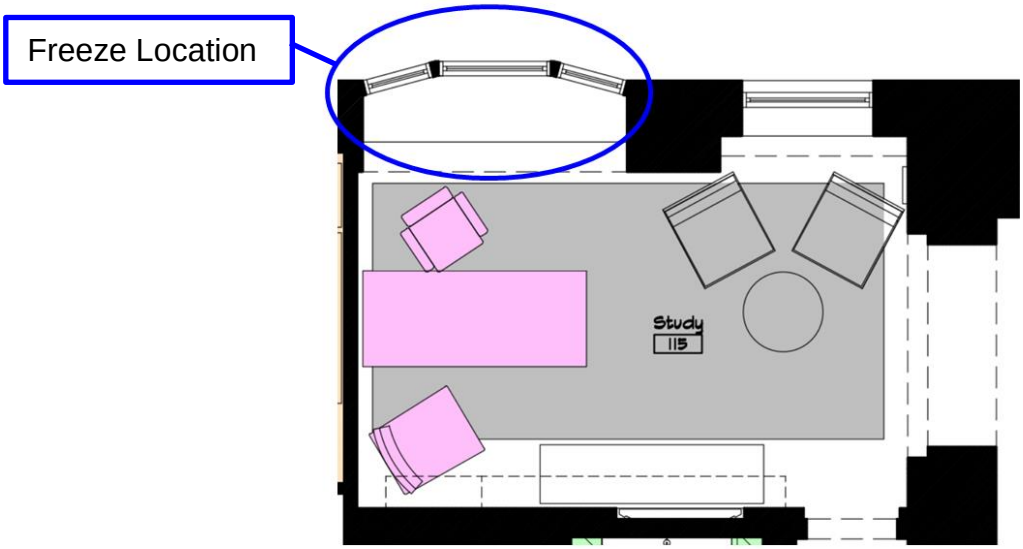


Figure B5

Details of Subject Freeze Location

Figure B6
Bay Window

Figure B7
Bay Window

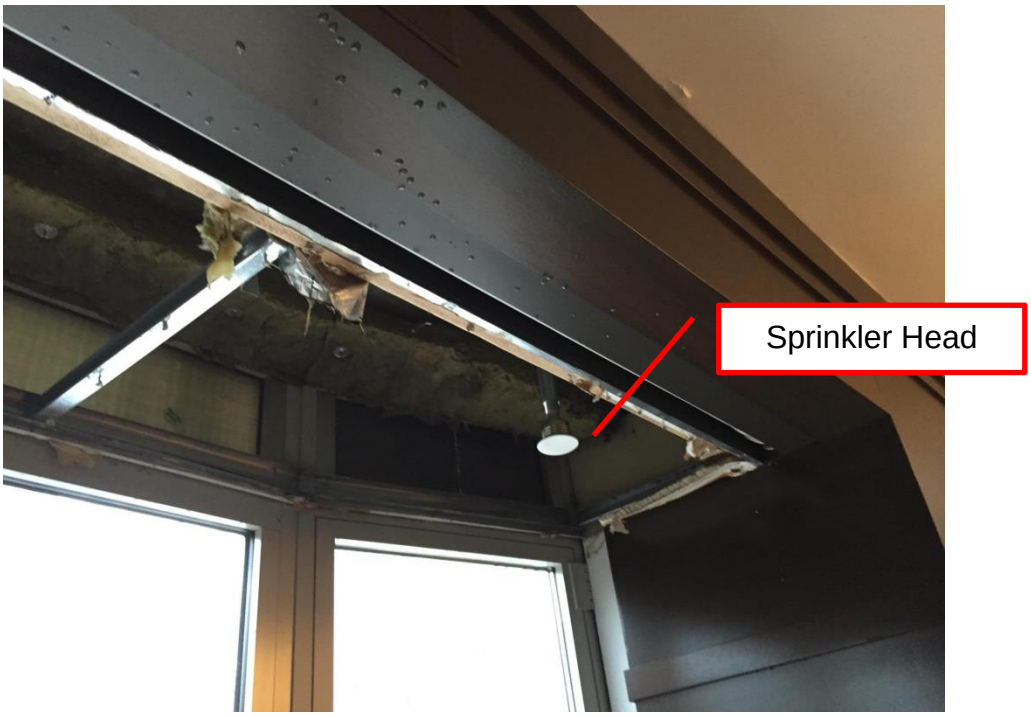


Figure B8
Sprinkler Head and View into Bay Window Cavity

Figure B9
Sprinkler Head and View into Subject Bay Window Cavity.

Note: The images above are based on the conditions two years after the incident. The subject bay window wall panels were removed, and locations of observed air infiltration were sprayed with foam insulation. Despite reasonable attempts to limit air infiltration, air infiltration persists (see figures below).

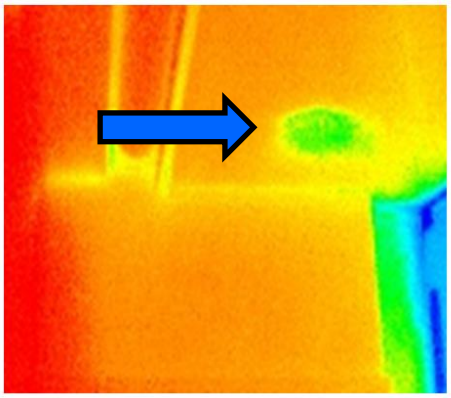


Figure B10
Infrared Image

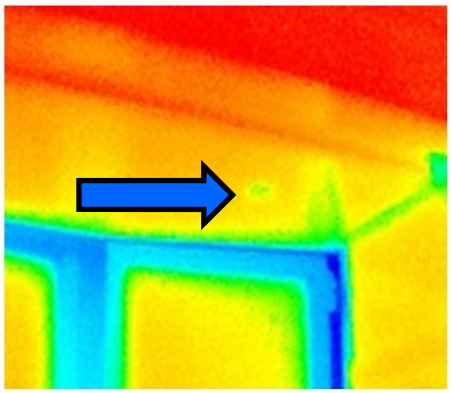


Figure B11
Infrared Image

Note 1: Infrared thermography, noticeable infiltration as indicated with blue arrow.

Note 2: Infiltration temperatures were measured and found to be 48°F with a 23°F outdoor temperature.

Infiltration Funneled through the Central Portal and Measured with Velometer @ 100 ft/min:

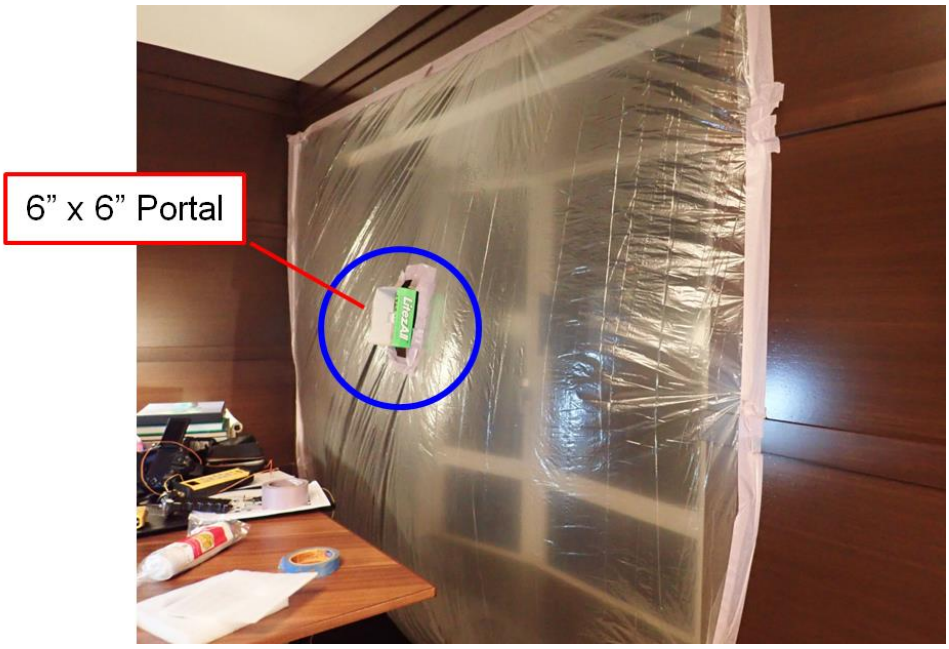


Figure B12

Window Leakage Testing.

CO_2 Tracer Gas Testing
(Readings recorded every 5 minutes)

Figure B13 **Figure B14** **Figure B15**

Figure B13: 20 lb CO_2 tank

Figure B14: Tare Weight

Figure B15: CO_2 monitor

Please note typical representation for exhaust fans, circulation fans, and fan-coils below

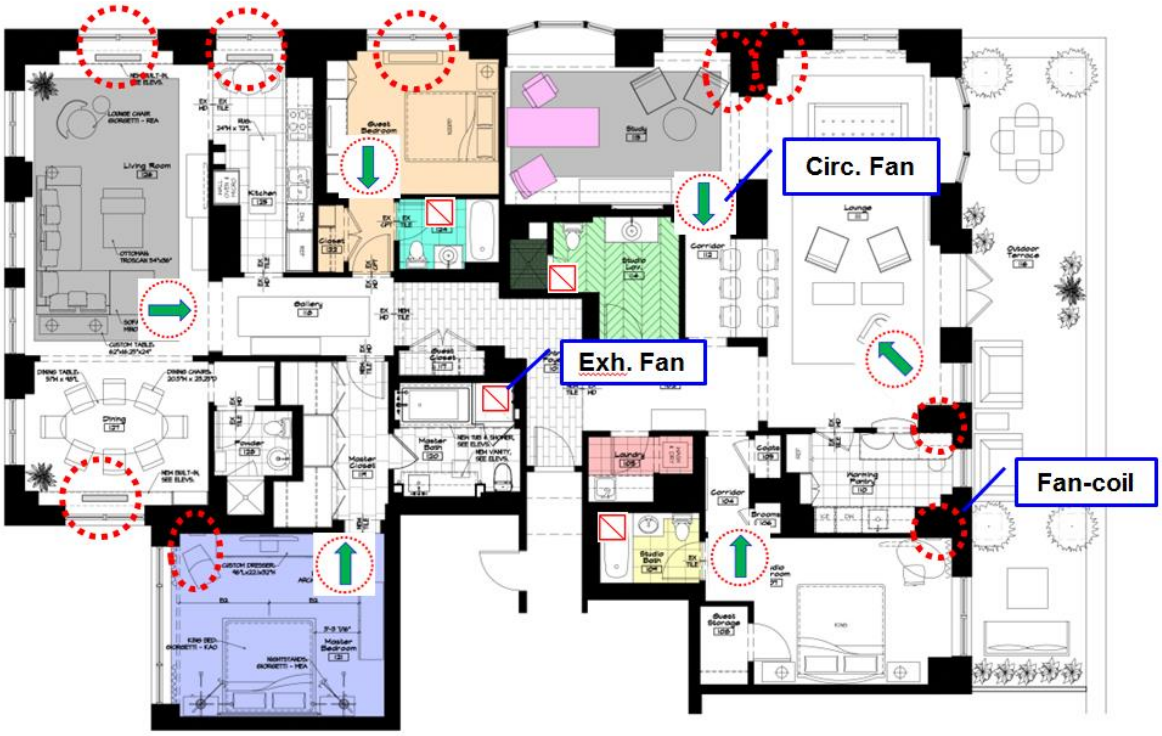


Figure B16
Floor Plan Details

Figure b17
Circulation Fan

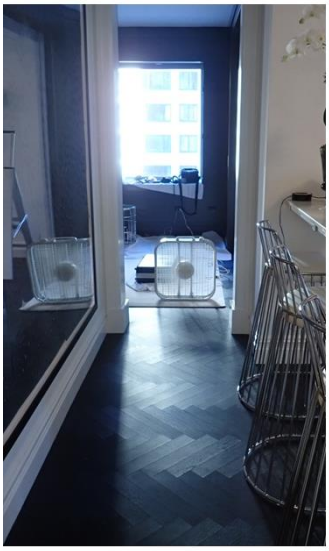


Figure B18
Circulation Fan

CO_2 Tracer Gas Observations and Data

- **Ambient Temp: 28°F**
- **Wind: 5-10 mph**
- **ΔP = 0.30" (H_2O)**
- **Toilet exhaust total: 120 cfm**

Floor Area:	**2,711**	**ft^2**	
Volume:	**24,399**	**ft^3**	
TIME	**CO_2**		**Infiltration**
(Min)	**(ppm)**	**ACH**	**(cfm)**
0	4,523		
5	4,209	0.918	373
10	3,966	0.771	314
15	3,667	1.011	411
20	3,427	0.887	361
25	3,189	0.949	386
30	2,948	1.042	424
35	2,776	0.818	333
40	2,604	0.876	356
45	2,427	0.971	395
	Ave:	**0.916**	

Figure B19

The subject heating/cooling settings were adjusted by rotating a controller knob with a warmer-cooler setting adjustment. Based on the manufacturer's specifications, the settings are as follows (see figure below):

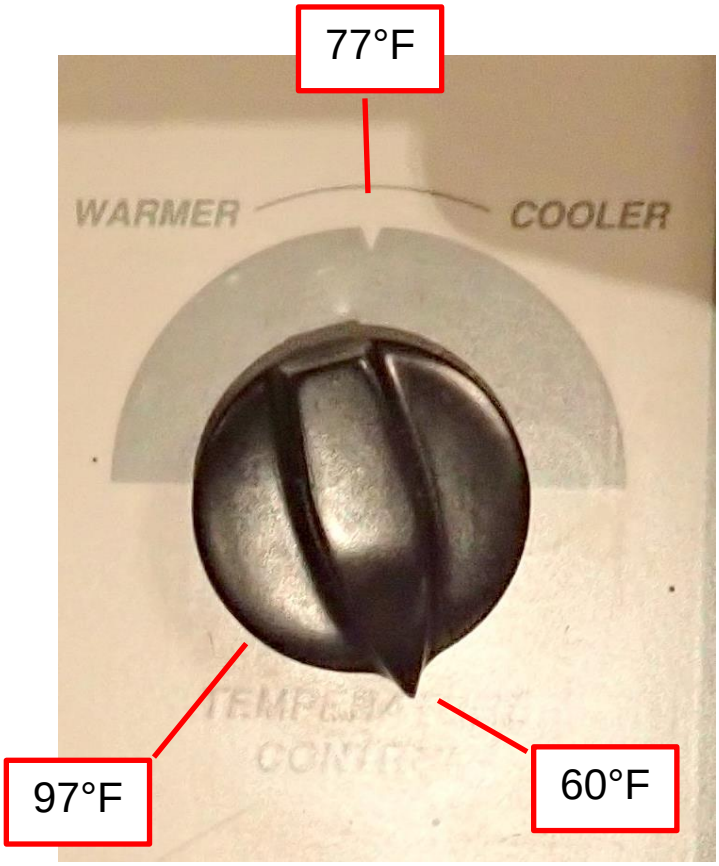


Figure B20
Typical Temperature Controller

In summary, the warmest temperature setting would be 97°F, and the coolest temperature setting would be 60°F.

The manufacturer's specifications for the subject temperature controller are shown in the next page below.

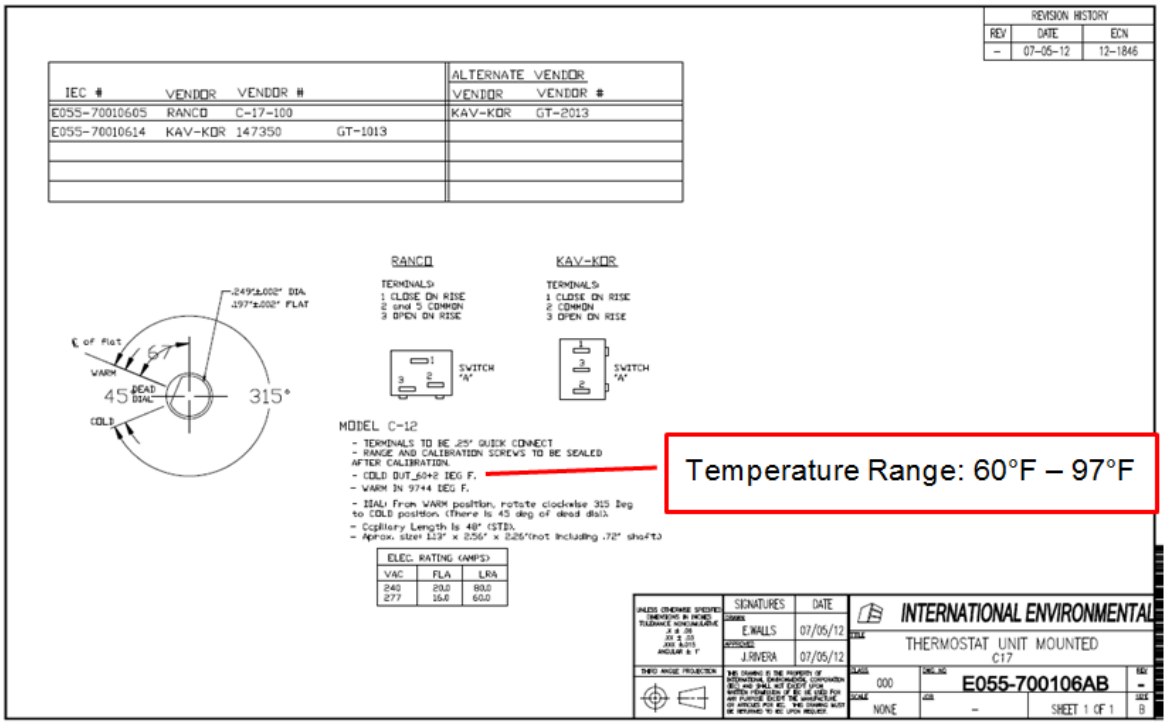


Figure B21
Manufacturer's Specifications for Subject Temperature Controller

It was further observed that, even when there was no call for heat, there was continuous circulation of hydronic heating fluid through header piping, resulting in a 3°F - 5°F temperature rise across each heating unit. The heat gain was also contributed by blower fan motors; each unit also has a 1/12 h.p. blower fan drawing 1.25 amps @ 120 VAC resulting in an internal heat gain of 150 watts or 409 Btu/hr per unit. It is further noted that, based on occupied temperatures of 72° for the abutter above and below, due to the high conductivity of concrete, heat transferred easily between the concrete slab floors.

A detailed heat load assessment of the outer perimeter walls/windows/doors (interior walls were not evaluated) was completed, documenting a heat load of 325 Btu/(hr -°F).

Based on detailed analysis, the modeled room temperature would be as noted in the figure below:

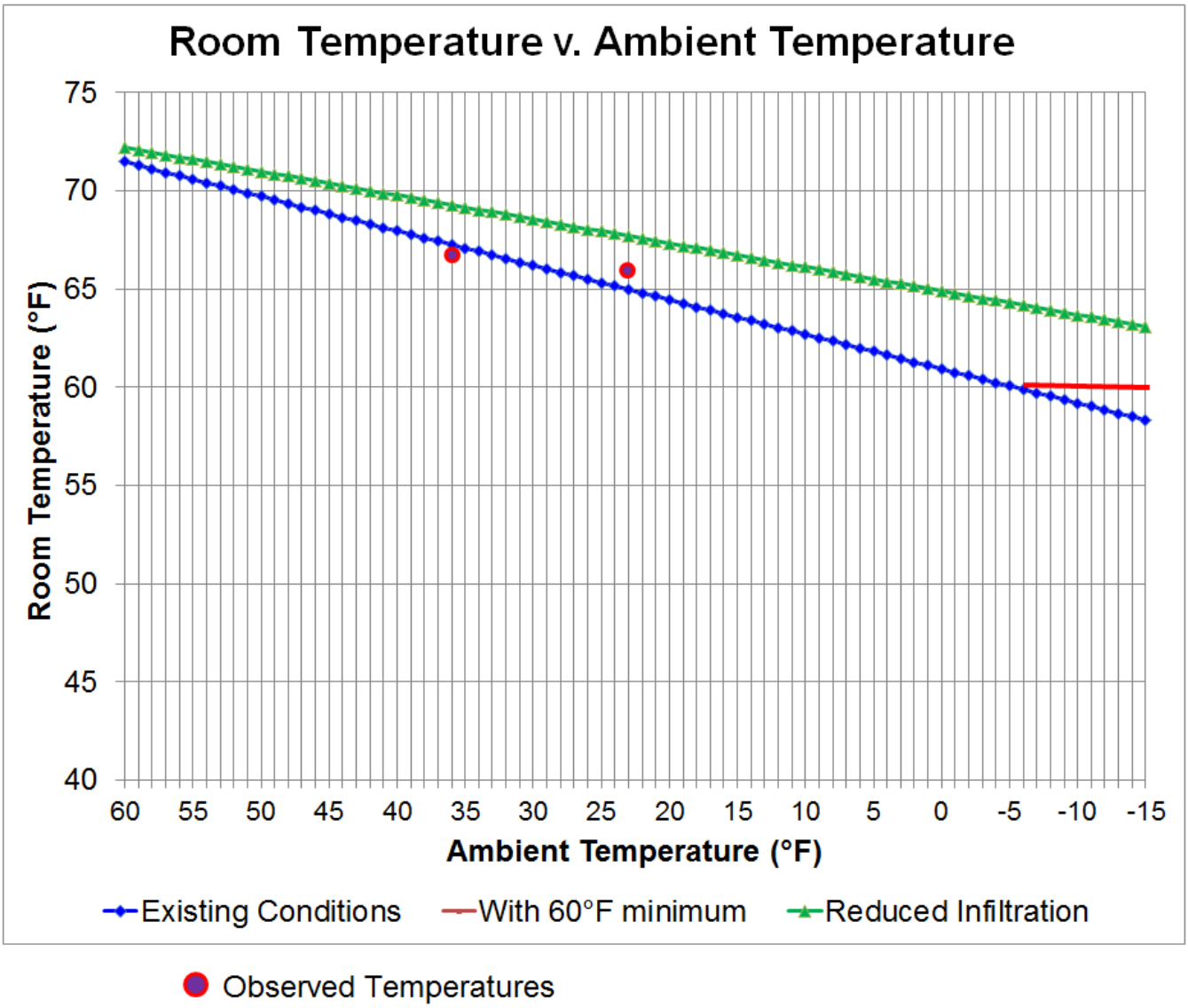


Figure B22
Room Temperature v. Ambient Temperature

Note: The green line represents the improved characteristics with control of bathroom exhaust and reduction in infiltration with the subject bay window assembly.

Case Study B Conclusions:

1) The cold air infiltration at the subject bay window contributed to the freeze event;

2) The overall air infiltration was above expected values and was approximately twice as much as average for residential structures.

3) The National Fire Protection Association (NFPA) standards **4.1.2.1** and **4.1.2.2** were not maintained; the code requires that fire sprinkler piping subject to temperatures below 40°F must be protected from freezing by approved methods.

4) The internal heat gain from the eight hydronic unit heaters, as well as infiltration and heat gain from the floors above and below, would only allow the subject unit to reach temperatures of 58°F; the minimum temperature settings of the thermostats was 60°F.

5) The subject bay window was located at the very bottom of the architectural section of the northern side of the building. The ascending bay windows positioned from the lower to the upper floors created a condition for stack effect resulting in excess infiltration.

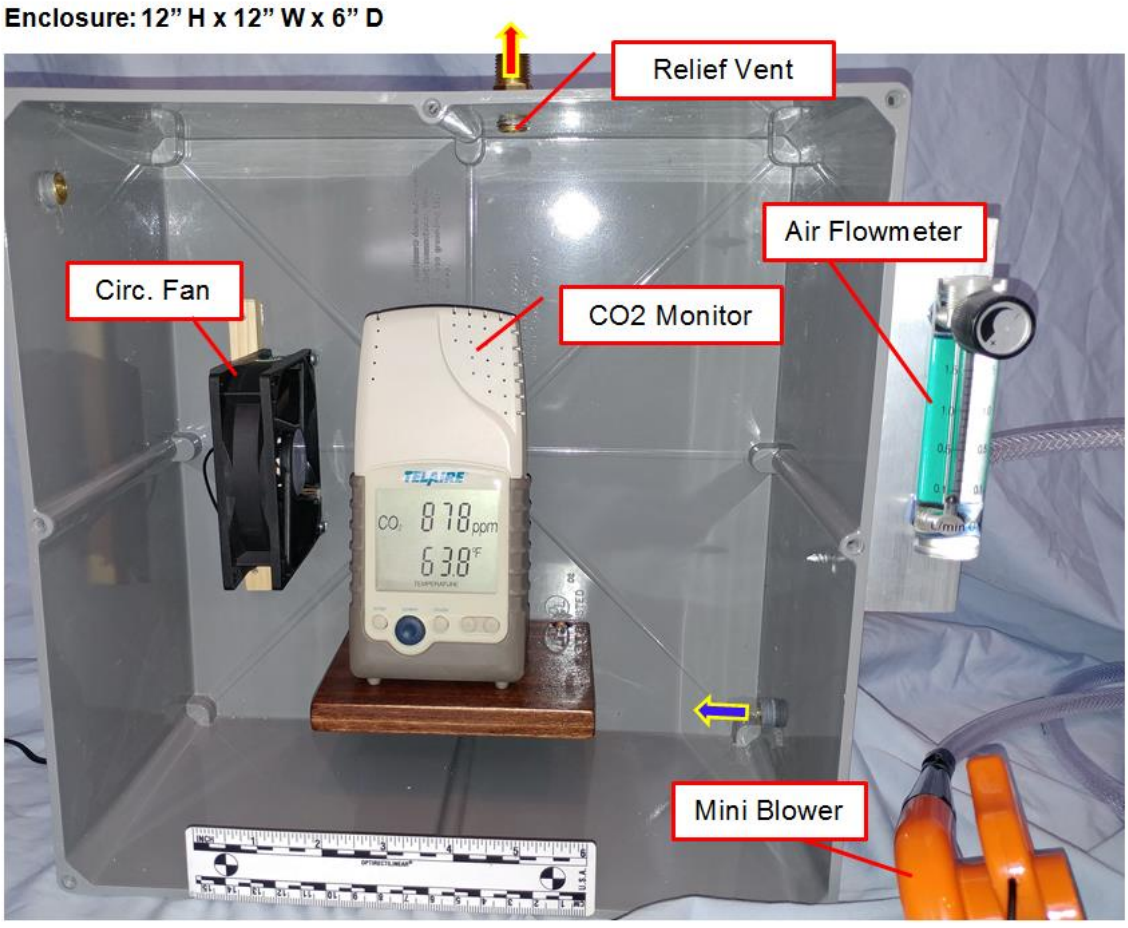
6) The subject bay windows need to have improved weatherization; the air leakage from the subject bay window has air infiltration that is more than double established standards.

7) The fire sprinkler piping should be traced with resistive electric heat tapes in a manner consistent with good engineering practice to avoid further freeze incidents to comply with NFPA 25 standards and accepted standards of good engineering practice.

Case Study C

Laboratory Test Apparatus

A 12” x 12” x 6” gasketed PVC electrical enclosure was utilized to determine air infiltration.



[Note: Lexan cover removed for photographic clarity purposes.]

Analysis using Discrete Equations and Iterative Numerical Techniques

In order to model the change in concentration of carbon dioxide in real time, the volume of the enclosure was included in the analysis. The blending of air and carbon dioxide is documented by a carbon balance see **Figure 3** below:

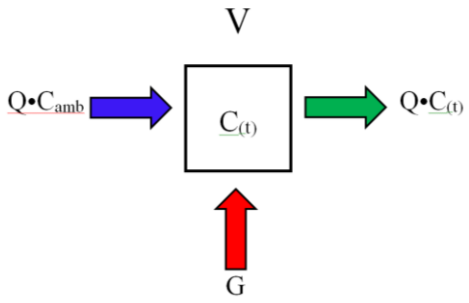


Figure 8
Carbon Dioxide Balance

$$\Sigma CO_2 = Q \bullet C_{amb} + G + V \bullet C_{(t)} - Q \bullet C_{(t)} \qquad \text{(Eq. \#4)}$$

CO_2 = Concentration: 1 ppm = cf of CO_2 per 1,000,000 cf of air
G – cf of CO_2 per time interval; i.e.: 1 MET = 0.00733 cf CO_2 per minute

1 MET = Metabolic human activity level for a typical 70 kg person

(t) = present interval iteration

V= Enclosure Volume

$$C_{t+1} = \frac{V \cdot C_t + G + Q \cdot C_{amb} - Q \cdot C_t}{V} = \frac{\Sigma CO2}{V} \qquad \text{(Eq. \#5)}$$

	H	L	W		
	(in)	(in)	(in)		
	12	20	6		
VOLUME	0.83	CF			
Camb	600	PPM			
	#people				
MET	0	0.00733	0.0	cf/min of CO_2	
Interval:	1	minutes			
			0.0	cf/interval of CO_2	
TIME	Data		Air Flow	Air Flow	Air Flow
(min)	(ppm)	ACH	(Cf/hr)	(Cf/min)	(liter/min)
0	5,844				
1	5,599	2.803	2.34	0.043	1.22
2	5,371	2.737	2.28	0.042	1.19
3	5,199	2.163	1.80	0.033	0.94
4	4,990	2.727	2.27	0.042	1.18
5	4,838	2.077	1.73	0.032	0.90
				Ave:	1.09

Summary Conclusions:

1) Three case studies were evaluated that utilized CO_2 for tracer gas testing:

    a) Case Study A, 60,000-square-foot Children's Recreational Facility
    b) Case Study **B**, High Rise Condominium Unit
    c) Case Study C, Laboratory Apparatus

2) The use of CO_2 as a tracer gas was implemented successfully and allowed a definitive determination of air infiltration rates. CO_2 was demonstrated to be an effective, economical, and safe alternative in comparison to other tracer gases.

3) The above air infiltration testing was completed in some cases along with the use of an infrared camera, and air balance instrumentation to provide additional information to meet engineering/investigation goals to document air infiltration through building structural components.

4) Each of these cases could have initially appeared to be explained by substandard materials, faulty equipment installation, or simple misfortune. However, further forensic engineering investigation using infrared camera imagery, CO_2 tracer gas testing, and utilization of weather data enabled a more specific opinion.

5) In each of the cases cited here, the use of CO_2 as a tracer gas was implemented successfully and allowed a definitive and accurate determination of air infiltration rates. CO_2 was demonstrated to be an effective, economic, and safe alternative to other tracer gases and a crucial tool to measure air infiltration and reveal

information that might be used to prevent damage and avoid costly subsequent repairs. CO_2 particularly lends itself to tracing air infiltration as it is non-toxic, readily available, and widely acceptable to building owners. Additionally, when properly administered, CO_2 tracer gas testing has been shown to be **90%-95%** percent accurate in determining the rate of air infiltration. Once the air infiltration has been determined, it is also possible to determine the rate of accumulation or decay of toxic gases such as carbon monoxide or other air contaminants. In addition, the determination of air infiltration rates can provide valuable information to building owners of excess air infiltration and energy consumption.

6) Calculation of "***Air Change Rate***" based on ASTM E741-11, section **8.5.3** will only be accurate if:

   a) Internal generation rate is at or near zero;

   <u>AND</u>

   b) If ambient concentrations of tracer gas are at or near zero;

   Since when using CO_2 "a" and/or "b" above would be affirmative then the calculation of mean air change rate $\bar{A}$ using Equation (2) would provide an erroneous value:

$$\bar{A} = [\ \ln C(t_2) - \ln C(t_1)\ ]/(t_2 - t_1) \qquad (2)$$

A modified equation has been demonstrated to be accurate if there is no self-generation:

$$\bar{A} = \left[\ \ln[C(t_1) - C_{amb}] - \ln[C(t_2) - C_{amb}]\right]/(t_2 - t_1) \qquad \text{ASTM E741 Equation (ii)}$$

7) The discrete iterative analysis using discrete equations #5 - #10 has been demonstrated to be accurate and directly account for internal CO_2 generation of tracer gas as well as ambient concentrations of tracer gas.

8) The ASTM E741 Equation (ii) can yield large errors exceeding 100% when there is self-generation.

9) Equation #5 along with Equation #10 even with high levels of self-generation ranging from 1-10 MET determines air change rates errors of less than **4**%.

10) ACH errors are significantly reduced when interior CO_2 levels are increased to 4,000 – 5,000 ppm, typical accuracy values would be 90% - 95%.

References

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2. Metabolic equivalents (METS) in exercise testing, exercise prescription, and evaluation of functional capacity; M Jetté, [1] K Sidney, G Blümchen, 1990.

3. ASTM E741, “Standard Test Method for Determining Air Change in a Single Zone by Means of a Tracer Gas Dilution.” 2017.

4. ASTM D6245, “Standard Guide for Using Indoor Carbon Dioxide Concentrations to Evaluate Indoor Air Quality and Ventilation,” 2017.

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